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Citation Weighting, Patent Ranking, and Apportionment of Value for Standard-Essential Patents

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A critical question repeatedly arises in litigation over the infringement of standard-essential patents (SEPs): What is an intellectually rigorous methodology for apportioning, across the various patents practiced in a multicomponent product, the value that the patents contribute to each enabling technology that gives the multicomponent product value? To address this question, it is necessary to derive an appropriate measure of the patents' value, relative to the value of other patents that are also essential to the standard. There exist many competing methodologies that purport to do so. We examine various patent-weighting methodologies that rely on forward patent citations to assign a patent's value. We show that one's choice of a given methodology is of secondary importance to the anterior question of whether to implement any patent-valuation methodology at all during the apportionment inquiry.

Legal experts, negotiators, and investment bankers often need to value a patent as part of an analysis or transaction. If comparable licenses for a patent

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exist, they might provide the best measure of that patent's value, because they represent freely negotiated contracts that provide information on the market price for the use of the licensed technology. However, comparable licenses might not exist—as would be the case if the SEPs are being licensed for the first time—and even those licenses that do exist might cover portfolios of patents and therefore would not provide valuations for individual patents.

When comparable licenses do not exist and there are no market transactions that can inform patent value, the value of the patent is what economists call a latent (or unobserved) variable.[1] When a variable cannot be directly observed, researchers can estimate its value by examining observable features that are correlated with the unobserved variable.[2] Just as patents themselves are often used to examine latent variables such as "knowledge transfer" or "innovation,"[3] a patent's features, such as the number of citations the patent has received, provide useful variables for estimating the value of that patent in enabling the technologies that its claims cover.

As part of the patent-valuation process, an expert must make several subjective decisions about how best to determine the relative value of the patents, given the available data and the objective of the analysis. For example, to value very young patents, an expert might need to formulate a predictive model to estimate the features that are most likely to emerge over time. Similarly, a portfolio of patents from very different technology fields might require an adjustment to control for features that differ across fields, whereas this adjustment might be less important if the patents concern similar technologies. Furthermore, a portfolio of patents from different periods might require a time adjustment, whereas a portfolio of patents from the same time period might not. To demonstrate how one could perform one or more of these adjustments, we use information from one particular technology standard—the load-reduced dual-inline memory module (LRDIMM) standard—and examine the patent values determined by various weighting techniques.[4] This analysis can help a researcher to decide which models best fit the needs of a particular case.

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1 *See* Charles Spearman, *"General Intelligence," Objectively Determined and Measured*, 15 Am. J. Psychol. 201 (1904) (providing an early exploration of latent variables); James J. Heckman, *Dummy Endogenous Variables in a Simultaneous Equation System*, 46 Econometrica 931 (1978) (providing an early use of latent variables in economics).

2 *See* sources cited in *supra* note 1; *see also* Manolo Romero Escobar, *Structural Equation Modeling: What is a Latent Variable?,* Analysis Factor, https://www.theanalysisfactor.com/what-is-a-latent-variable/.

3 *See* Adam B. Jaffe & Manuel Trajtenberg, Patents, Citations, and Innovations: A Window on the Knowledge Economy 1 (MIT Press 2002); *see also* Nathan Falk & Kenneth Train, *Patent Valuation with Forecasts of Forward Citations*, 12 J. Bus. Valuation & Econ. Loss Analysis 101 (2017).

4 This article extends our analysis of the LRDIMM standard presented in J. Gregory Sidak & Jeremy O. Skog, *Hedonic Prices and Patent Royalties*, 2 Criterion J. on Innovation 601, 604–08, 621–25 (2017).

In this article, we examine several patent-valuation methods that we have observed in the economic literature. Using those methods, as well as a method that we propose that weights forward citations according to how quickly the citations occur after the initial patent is granted, we construct measures of patent value for the patents declared essential to the LRDIMM standard. We find that all of the weighting measures produce similar levels of patent inequality, and that all weighted measures produce different levels of inequality from that of an unweighted measure in which all patents are assigned equal value. That is, a weighted measure produces a meaningfully different valuation of patents than an unweighted measure produces. We conclude that the choice of a particular weighting measure is of secondary importance to the decision to use *some* weighting method rather than none.

In Part I of this article, we briefly discuss the legal reasoning behind apportioning patent value, and the unique issues that arise for SEPs. In Part II, we briefly describe the LRDIMM standard and examine its development over time as new patents have been declared essential to practice the technology described in the standard. In Part III, we examine common methods of patent-citation valuation and calculate the weights placed on each patent in the LRDIMM standard using each method. In Part IV, we examine how the distribution of value among patents within the LRDIMM standard varies when using different weighting methods.

I. The Legal Requirement of Apportionment

Section 284 of the Patent Act requires that a court award patent damages that are "adequate to compensate for the infringement, but in no event less than a reasonable royalty for the use made of the invention by the infringer."[5] The Supreme Court has long held that, when a "patent is for an improvement, and not for an entirely new machine or contrivance, the patentee must show in what particulars his improvement has added to the usefulness of the machine or contrivance."[6] It emphasized that "[t]he patentee . . . must in every case give evidence tending to separate or apportion the defendant's profits and the patentee's damages between the patented feature and the unpatented features."[7] Failure to apportion damages to the appropriate value of the patented technologies would result in the overcompensation of the patent holder.

[5] 35 U.S.C. § 284.

[6] Garretson v. Clark, 111 U.S. 120, 121 (1884).

[7] *Id.*; *see also* City of Elizabeth v. Am. Nicholson Pavement Co., 97 U.S. 126, 138–39 (1877); Keystone Mfg. Co. v. Adams, 151 U.S. 139, 147–48 (1894).

The requirement to apportion damages to the value of the patented invention plays a particularly important role in litigation involving the infringement of patents practiced in a multicomponent product. In such cases, the expert witness on damages must use a methodology that will distinguish the value attributable to the patented invention from the value attributable to the product's noninfringing components.[8] Because SEPs are typically implemented in complex products that include many patented and unpatented components, apportionment is particularly relevant in patent cases involving infringing products that implement an industry standard.

In 2014, the Federal Circuit said in *Ericsson v. D-Link* that "[w]hen dealing with SEPs, there are two special apportionment issues that arise."[9] First, the "patented feature must be apportioned from all of the unpatented features reflected in the standard."[10] Second, the royalty "must be premised on the value of the patented feature, not any value added by the standard's adoption of the patented technology."[11] The Federal Circuit reasoned that those two steps "are necessary to ensure that the royalty award is based on the incremental value that the patented *invention* adds to the product, not any value added by the standardization of that technology."[12] In 2015, the Federal Circuit elaborated that the requirement to exclude any value added by the inclusion of the technology into a standard applies to all SEPs, regardless of whether they are subject to an obligation on the patent holder's part to offer to license the patents on fair, reasonable, and nondiscriminatory (FRAND) or reasonable and nondiscriminatory (RAND) terms to willing implementers of the standard.[13]

In our previous article, *Hedonic Prices and Patent Royalties*, we showed how comparing the prices paid for products incorporating the standard relative to an older generation of the standard could enable one to measure the value of that standard.[14] Here, we demonstrate different methods one could use to apportion the value of that standard to its constituent patents.

[8] Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1226 (Fed. Cir. 2014) (citing VirnetX, Inc. v. Cisco Sys., Inc., 767 F.3d 1308, 1326 (Fed. Cir. 2014)).

[9] *Id.* at 1232.

[10] *Id.*

[11] *Id.*

[12] *Id.* (emphasis in original).

[13] Commonwealth Sci. & Indus. Research Org. v. Cisco Sys., Inc., 809 F.3d 1295, 1304 (Fed. Cir. 2015). In 2018, the Federal Circuit reiterated: "If the product has other valuable features that also contribute to driving consumer demand—patented or unpatented—then the damages for patent infringement must be apportioned to reflect only the value of the patented feature." Power Integrations, Inc. v. Fairchild Semiconductor Int'l, Inc., No. 16-2691, slip op. at 20 (Fed. Cir. July 3, 2018).

[14] *See* Sidak & Skog, *Hedonic Prices and Patent Royalties*, *supra* note 4, at 611–20.

II. Standard-Essential Patents and the LRDIMM Standard

The technology analyzed in this article concerns certain standardized interfaces implemented on dual-inline memory modules (DIMMs) used primarily in enterprise servers. Server DIMMs interact with a server's central processing unit (CPU) to enable storage and access of information actively used to run operating systems and programs and perform numerous functions.[15] We have studied two specific types of server DIMMs standardized by the Joint Electron Devices Engineering Council (JEDEC): load-reduced dual-inline memory modules (LRDIMMs) and registered dual-inline memory modules (RDIMMs) using the fourth-generation double data rate (DDR4) dynamic random-access memory (DRAM) chipsets.

A. Standardized Memory Modules Used in Server Applications

JEDEC is an international standard-setting organization (SSO) with more than 250 member companies that jointly develop standards for the microelectronics industry, including standards for solid-state devices, integrated circuits, and electronic modules, such as memory modules.[16] JEDEC's DDR4 LRDIMM and RDIMM standards are widely implemented in memory modules sold by DRAM manufacturers such as Samsung, SK hynix, and Micron and typically purchased by original equipment manufacturers of servers, including Hewlett-Packard, IBM, and Dell.[17]

1. JEDEC's RDIMM, LRDIMM, DDR3, and DDR4 Standards

JEDEC's RDIMM and LRDIMM standards each specify unique interfaces for server DIMMs. JEDEC's RDIMM standard introduces a single register on the memory module that buffers the read and write commands between the DRAM chipsets and the server's memory controller.[18] The addition of the register on the memory module enables the user to increase the server's

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15 *See, e.g.*, *The Role of Memory in Your Computer*, Crucial, http://www.crucial.com/usa/en/support-what-does-computer-memory-dram-do.

16 JEDEC, JEDEC Manual of Organization and Procedure § 1.2, at 1 (July 2015), http://www.jedec.org/sites/default/files/JM21R.pdf [hereinafter JEDEC Manual]; *see also About JEDEC*, JEDEC, https://www.jedec.org/about-jedec.

17 *See, e.g.*, *Products-DRAM*, Samsung, http://www.samsung.com/semiconductor/products/dram/; *Products-DRAM-Module-Product List*, SK hynix, https://www.skhynix.com/products.do?lang=eng&ct1=36&ct2=42&ct3=&rk=&sk=#tg02; *Products-DRAM Modules*, Micron, https://www.micron.com/products/dram-modules; Press Release, SK hynix Inc., Hyundai Electronics Actively Supplying DDR SDRAM Modules to Major PC Makers (Mar. 29, 2001), https://www.skhynix.com/eng/pr/pressReleaseView.do?seq=1035&offset=1&searchWord=Dell.

18 *See, e.g.*, Johan De Gelas, *LRDIMMs, RDIMMs, and Supermicro's Latest Twin*, AnandTech (Aug. 3, 2012), http://www.anandtech.com/show/6068/lrdimms-rdimms-supermicros-latest-twin/2.

memory capacity relative to an unbuffered DIMM (UDIMM).[19] By adding nine distributed buffers to the DRAM module, JEDEC's DDR4 LRDIMM standard further increases the server's memory capacity while maintaining or even increasing the server's memory bandwidth.[20] In other words, relative to memory modules practicing JEDEC's earlier RDIMM standard, a DDR4 LRDIMM product enables greater memory capacity without impeding the server's ability to operate at its highest system speed.[21] DDR4 LRDIMM products thus offer better server performance at a high memory capacity than any other server DIMM.[22]

Meanwhile, apart from the transition from the RDIMM standard to the LRDIMM standard, advances in DRAM technology have also improved the performance of memory modules. DDR4 is JEDEC's standard for the latest generation of DRAM products. (JEDEC's standard for the next generation of DRAM, DDR5, is currently in development.[23]) Relative to the earlier DDR3 products, DDR4 products offer a higher memory bandwidth interface and increased energy efficiency.[24]

2. *JEDEC's Patent Policy and RAND Commitment*

Member companies that participate in JEDEC's standard-setting process must adhere to the SSO's Manual of Organization and Procedure, which defines JEDEC's patent policy and its RAND commitment.[25] JEDEC's patent policy requires members to disclose to JEDEC the patents potentially essential to practice the discussed standard and to memorialize their willingness (or unwillingness) to offer to license those SEPs on RAND terms in a license disclosure that JEDEC subsequently makes available to

19 *Id.*

20 *See* Douglas Malech, *LRDIMM vs RDIMM: Signal Integrity, Capacity, Bandwidth*, EDN Network (Aug. 1, 2014), http://www.edn.com/design/designcon/4432983/LRDIMM-vs-RDIMM--Signal-integrity--capacity--bandwidth; *Technical Brief—LRDIMMs*, Kingston Tech., https://www.kingston.com/us/memory/resources/lrdimms.

21 *See* Malech, *LRDIMM vs RDIMM: Signal Integrity, Capacity, Bandwidth*, *supra* note 20; Douglas Malech, Sameer Kuppahalli, Ryan Baxter & Eric Caward, *DDR4 LRDIMMs Let You Have It All* 3–4, IDT & Micron (2016) [hereinafter IDT & Micron White Paper], https://www.idt.com/document/whp/idt-ddr4-lrdimms-let-you-have-it-all?language=en.

22 IDT & Micron White Paper, *supra* note 21, at 2.

23 *See, e.g.*, Press Release, JEDEC, JEDEC DDR5 & NVDIMM-P Standards Under Development (Mar. 30, 2017), https://www.jedec.org/news/pressreleases/jedec-ddr5-nvdimm-p-standards-under-development; *DDR5 DIMM Chipset*, Rambus, https://www.rambus.com/memory-and-interfaces/server-dimm-chipsets/ddr5-dimm-chipset/ ("DDR5 is a working DRAM standard being created by JEDEC and is the next evolutionary step from DDR4."); Anton Shilov, *Cadence and Micron Demo DDR5-4400 IMC and Memory, Due in 2019*, AnandTech (May 3, 2018), https://www.anandtech.com/show/12710/cadence-micron-demo-ddr5-subsystem ("The final DDR5 specification is expected to be published by JEDEC this summer.").

24 *See DDR4 SDRAM—Overview*, SK hynix, https://www.skhynix.com/products.do?lang=eng&ct1=36&ct2=37&rc=com.

25 JEDEC Manual, *supra* note 16, § 8.2, at 24.

other members.[26] JEDEC provides its members online access to all submitted license assurances and the information conveyed therein, including the declared-essential patents, the patent assignee, and the relevant standard to which the patent owner believes the patents are essential.[27] By reviewing these lists of patents, we identify which patents have been declared essential to each JEDEC standard.

JEDEC's patent policy explains that "each Committee Member, as a condition of Participation, agrees to offer to license on RAND terms, to all Potential Licensees, such . . . Member's Essential Patent Claims."[28] Similarly, the license assurance that a member gives to JEDEC contains the following statement concerning the making of a RAND offer to a potential licensee: "A license will be offered to applicants desiring to utilize the license for the purpose of implementing the JEDEC Standard under reasonable terms and conditions that are demonstrably free of any unfair discrimination."[29]

The proper interpretation of JEDEC's RAND commitment has been subject to dispute. New York law controls the interpretation of JEDEC's patent policy and the precise obligations arising from a member's RAND commitment to JEDEC.[30] In the 1023 Investigation, *Certain Memory Modules and Components Thereof*, Chief Administrative Law Judge (ALJ) Charles Bullock of the U.S. International Trade Commission (ITC), in the public version of his Initial Determination on Violation of Section 337 and Recommended Determination on Remedy and Bond, found that, based on the specific facts of the case, the complainant's RAND commitment to JEDEC was too ambiguous to constitute an enforceable contract.[31] Chief ALJ Bullock observed that "the parties make no attempt to analyze Complainant's RAND obligations according to the New York state law."[32] He found the respondents' failure to do so "particularly troubling," because, "under New York law, '[a] court cannot enforce a contract unless it is able to determine what in fact the parties have agreed to,' . . . and '[i]f an agreement is not reasonably certain in its material terms, there can be no legally enforceable contract.'"[33] Chief ALJ Bullock reasoned that, "given that the undersigned cannot determine what

26 *Id.* § 8.2.2.1, at 24.

27 JEDEC members can access these assurances by logging on to JEDEC's member website, clicking on the "Patents" tab, and downloading an Excel spreadsheet that itemizes the submitted assurances and includes embedded hyperlinks to a PDF download for each document. *Patent Letters*, JEDEC, https://members.jedec.org/PatentLetters/PatentLetters.xls.

28 JEDEC Manual, *supra* note 16, § 8.2.4, at 26.

29 *See, e.g.*, JEDEC, License Assurance/Disclosure Form [hereinafter JEDEC License Assurance], http://www.jedec.org/sites/default/files/License_Assurance-Disclosure_Form_20150710.pdf.

30 JEDEC Manual, *supra* note 16, § 8.2.10, at 29.

31 Inv. No. 337-TA-1023, slip op. at 195 (USITC Nov. 14, 2017) (Initial Determination—Public Version). In the 1023 Investigation, Sidak was an expert economic witness for the complainant, Netlist Inc., and Skog performed economic analysis in support of Sidak's testimony.

32 *Id.* at 194.

33 *Id.* (quoting Carione v. Hickey, 20 N.Y.S.3d 157, 158 (App. Div. 2015)).

exactly the RAND commitment entails in terms of acceptable licensing terms, New York law appears to require that the agreement be considered unenforceable."[34]

The complainant and respondents in the 1023 investigation subsequently petitioned the Commission to review Chief ALJ Bullock's Initial Determination.[35] The Commission agreed to review in part Chief ALJ Bullock's Initial Determination, and it affirmed his finding of no violation of section 337.[36] However, the Commission offered no opinion on other parts of Chief ALJ Bullock's Initial Determination (including on issues concerning the RAND obligation), and it subsequently terminated the investigation.[37]

Chief ALJ Bullock's ruling does not alter our analysis. As we explained in Part I, the Federal Circuit said that the requirement to apportion damages (and thus to isolate the value of the standard from the value of standardization) applies regardless of whether the SEPs are encumbered by a RAND commitment.[38]

B. Identifying Patents Declared Essential to JEDEC's LRDIMM Standard

Companies, universities, and other participants submit letters to JEDEC declaring the patents that they consider essential to practice the LRDIMM standard.[39] In each letter, the submitter can identify one or more patents so that a third party that wishes to practice the standard can acquire licenses to all of the relevant intellectual property. To identify the patents declared essential to the LRDIMM standard, we first analyzed data on license assurances that patent holders submitted to JEDEC disclosing their patents as essential to the LRDIMM standard. Then, using each SEP holder's submitted keywords describing the relevant technology in that database, we created a list of keywords with which to search JEDEC's database of submitted license assurances. The keywords that we used for identifying license assurances that disclosed patents essential to the LRDIMM standard were Load decoupling DIMM, LD-DIMM, LD DIMM, LDDIMM, LR-DIMM, LR DIMM, LRDIMM, Load reducing DIMM, Reduced load DIMM, and

34 *Id.* at 195.

35 Certain Memory Modules and Components Thereof, and Products Containing Same: Commission Determination to Review-in-Part an Initial Determination Finding No Violation of Section 337; On Review, to Take No Position on One Issue; Affirmance of the Finding of No Violation and Termination of the Investigation, Inv. No. 337-TA-1023, 83 Fed. Reg. 3023, 3024 (Jan. 22, 2018), https://www.gpo.gov/fdsys/pkg/FR-2018-01-22/pdf/FR-2018-01-22.pdf ("On November 27, 2017, complainant and respondents petitioned for review of the final ID.").

36 Notice of a Commission Determination to Review-in-Part an Initial Determination Finding No Violation of Section 337; On Review, to Take No Position on One Issue; Affirmance of the Finding of No Violation and Termination of the Investigation, Certain Memory Modules and Components Thereof, and Products Containing Same, Inv. No. 337-TA-1023 (USITC Jan. 16, 2018).

37 *Id.*

38 Commonwealth Sci. & Indus. Research Org. v. Cisco Sys., Inc., 809 F.3d 1295, 1304 (Fed. Cir. 2015).

39 JEDEC Manual, *supra* note 16, § 8.2.3, at 25; *see also* JEDEC License Assurance, *supra* note 29.

Reduced load DIMMS. Capitalization and hyphenation do not affect the results of this keyword identification.

This procedure identified license assurances that had declared patents as being essential to the LRDIMM standard. We then inspected summaries of each license assurance, which JEDEC maintains, to ensure that our search successfully identified the relevant license assurances and that the terms that we used related to the terms that SEP holders themselves used to describe the technology covered by their declared essential patents. If a patent were erroneously excluded from this analysis, the exclusion would need to be systematically biased for or against a particular SEP holder submitting license assurances to bias our apportionment results. Otherwise, the omission would be harmless error.

We then compiled a list of those declared SEPs that enabled us to identify a master list of 53 U.S. patents declared essential to JEDEC's LRDIMM standard.[40] We further verified those SEPs with publicly available data from the PatentsView database compiled by the USPTO, and we identified additional data regarding those SEPs.[41] The USPTO data include variables for an SEP's features, claims, grant and expiration dates, and forward citations (which measure when the patent of interest is cited as relevant for a later patent, either by the later patent's applicant or by a patent examiner).

Using the patents that we identified as being declared essential to the LRDIMM standard, we examined the LRDIMM standard's development over time. This analysis is backward-looking in that we examine patents that were declared essential as of January 2018, and we do not examine any issued patents or any patent applications that could be declared essential to the LRDIMM standard after that date. However, these data would allow us to examine the shape of the distribution of LRDIMM SEPs in the past. Here, we provide descriptive statistics of the history of the LRDIMM standard.

The earliest patent application for an LRDIMM SEP was filed on May 22, 2001.[42] The patent was issued on December 24, 2002.[43] This patent was first cited on March 1, 2005, which is slightly more than two years after its issuance.[44] It became the first patent declared essential to the LRDIMM standard on August 20, 2008.[45] As of January 2018, the most recent patents

[40] As of the publication of this article, three of the 53 patents have expired. Those expired patents include U.S. Patent Nos. 6,820,181, 7,349,277, and 7,366,920.

[41] *Data Query*, PatentsView, http://www.patentsview.org/query/. PatentsView is "a patent data visualization and analysis platform intended to increase the value, utility, and transparency of prototype US patent data," an initiative supported by the USPTO's Office of Chief Economist. *FAQs—What Is PatentsView?*, USPTO, http://www.patentsview.org/api/faqs. html. The data are "sourced from USPTO-provided text and XML data on published patent applications (2001–present) and granted patents (1976–present)." *Id.*

[42] U.S. Patent No. 6,498,766 (filed May 22, 2001).

[43] *Id.*

[44] U.S. Patent No. 6,862,248 (filed Dec. 16, 2003).

[45] Samsung Electronics License Assurance/Disclosure Form (JEDEC Ref. No. 257) (Aug. 20, 2008).

to be declared essential to the standard were declared on April 7, 2016.[46] As of January 2018, the most recent SEP was issued on September 8, 2015, although no other patents cite to it.[47] Thus, although the development of JEDEC's LRDIMM standard occurred over 8 years—from the declaration of the first SEP in 2008 to the final declaration in 2016—the development of the LRDIMM technology itself began in 2001.

Typically, some time passes between the moment when a patent is issued and the moment when that patent is first cited by another patent, although the shortest interval that we observe in our database is 116 days.[48] The average interval from patent issuance until the first citing patent is issued is 1,848 days (or 5.06 years),[49] and the median interval is 1,757 days (or 4.81 years).[50] There is also variation in the length of time between the moment when a patent application is submitted and the moment when the patent is issued. The shortest interval that we observe is 92 days,[51] whereas the longest interval is 1,657 days (or 4.54 years).[52] The average interval before the application is issued is 856 days (or 2.35 years),[53] whereas the median interval is 822 days (or 2.25 years).[54] Figure 1 shows a timeline of the LRDIMM standard's development.

46 Netlist, Inc. License Assurance/Disclosure Form (JEDEC Ref. No. 518) (Apr. 7, 2016); Netlist, Inc. License Assurance/Disclosure Form (JEDEC Ref. No. 519) (Apr. 7, 2016); Netlist, Inc. License Assurance/Disclosure Form (JEDEC Ref. No. 520) (Apr. 7, 2016); Netlist, Inc. License Assurance/Disclosure Form (JEDEC Ref. No. 521) (Apr. 7, 2016).

47 U.S. Patent No. 9,128,632 (filed July 27, 2013).

48 *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

49 *Id.*

50 *Id.*

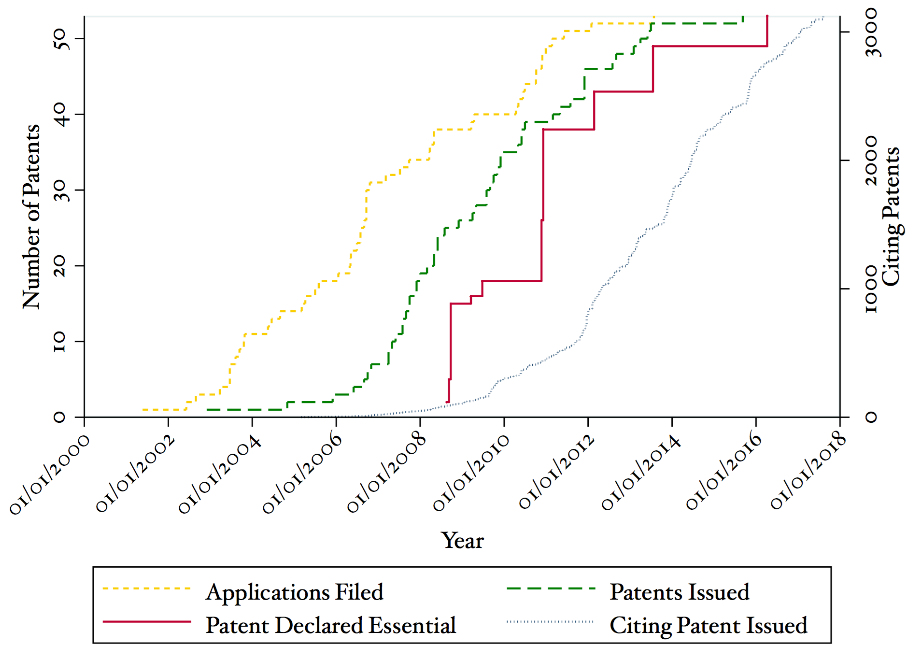
51 *Id.*

52 *Id.*

53 *Id.*

54 *Id.*

Figure 1. Timeline of the Development of the LRDIMM Standard



Source: *Patent Letters*, JEDEC, *supra* note 27; *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

The vertical axis shows the number of declared-essential patents filed, issued, and granted. The relative height of each line between the horizontal axis and the top of the graph represents the relative portion of the 53 currently declared-essential patents filed, issued, and granted. The right-hand vertical axis in the graph shows the number of citations; the line's relative height shows the proportion of currently observed citations that were made at a particular date indicated on the horizontal axis. There is less vertical distance between the lines over time as the patents are filed, issued, and then declared. That is, a greater portion of the patents eventually declared essential to the standard have achieved their granted status as time progresses. The height of the graph depends on the number of patents declared. In particular, the relative smoothness of the lines indicating patent filing, patent issuance, and patent citations contrasts with the stepwise shape of the line indicating patent declarations. This pattern suggests that companies often declare multiple patents simultaneously, rather than declare individual patents at different times during the standard's development.

Consequently, one must use caution when apportioning a standard's value *during* the standard-setting process, as the results of that apportionment are likely to vary depending on the specific moment at which one analyzes the

SEPs. For example, a patent's estimated relative contribution to a standard might decrease after a new batch of patents is declared essential to that standard. Once most patents essential to the LRDIMM standard have been declared, however, the distribution of those SEPs' filing, issuance, and citation dates appear to be more uniformly distributed over time.

III. Approaches to Patent Valuation

Various methods of apportioning value to individual patents exist, and the researcher often must make a subjective choice about which method is most appropriate in a given case. This choice will be determined by the availability of appropriate data, the specific facts of the case, and the objectives of the research.

It also bears emphasis that, in our experience, when negotiating a license agreement for a patent portfolio, parties typically do not assess the value of each individual patent but focus instead on the value of the licensed patent portfolio as a whole. However, a patent holder that is enforcing a patent in court cannot feasibly seek to enforce all of its patents simultaneously. Such is the case when the patent holder's portfolio includes hundreds of patents, and the patent holder is forced by the constraints of the judicial process to litigate only a subset of its patents (and, even then, only a subset of the claims of those patents). In that case, the court might need to apportion damages only for the infringement of the patents (or the particular patent claims) that, through litigation, have been found to be valid and infringed. Thus, the valuation of individual patents in a portfolio is typically motivated by the need to answer a question that a legal proceeding has assigned to the finder of fact. Although the valuation of individual patents might be associated with a larger degree of error relative to the valuation of a patent portfolio, the relevant statistical question is whether the errors are biased in one direction, such that the results are unhelpful, or whether the errors are unbiased and thus tend to cancel out one another.

In this part, we explain the assumptions behind various citation-counting methods, we describe the situations in which they might be useful, and we examine the effects of employing them, using the SEPs necessary to practice the LRDIMM standard as an example data set.

We examine valuation methods that treat patents as similar based on observed forward citations. An alternative approach is to examine the features of the patent and predict how many citations a patent is likely to receive. As with any prediction, the fundamental assumption underlying that method is that there is some pattern in past data that we believe will continue to occur in the future. That is, we would model some pattern observed in the past and apply that pattern to current values. This method effectively assumes that current patents will behave the same as past patents. In this way, a prediction method treats observations at different points in time as though they were at the same point in time.

To compare patent value across time, one might need to weight citation counts to control for changes in citation patterns. To control successfully for changes in those trends, one must identify the pattern that is the most stable over time and therefore the most likely to continue. There also likely exists a tradeoff between the accuracy of describing past behavior and the likelihood that a pattern will continue into the future. This tradeoff is sometimes described as "overfitting," which occurs when one describes past behavior so precisely that one is unlikely to make accurate predictions of future behavior.[55]

Other adjustment techniques define a "cohort" of similar patents, either based on the time when they were granted or on the technologies covered by the patents' claims. Patents are then compared to other patents in their cohort on measures such as citations to determine their relative rank. The cohort-adjusted patents can then be compared with each other. We add a method that applies an increasing penalty term to citations that occur a longer time after a patent has been granted.

We gather patent and citation data from the PatentsView website, which is managed by the U.S. Patent and Trademark Office (USPTO) and offers information on patent-grant date, forward-citing patents as well as their grant dates, and the technology field of each patent.[56] We use these measures to create seven measures of patent value, including a count of patents, a count of citations, a count of recent citations, an age-weighted count of citations, and a measure of the number of citations that a patent has, relative to its age-technology field cohort. We examine the distribution of the standards' value share using each measure of an individual patent's relative value.

A. Patent Counting

The simplest method of patent valuation is an equal-share patent count. This method assigns an equal value to all patents declared essential to a standard. Each company's share of the value of the standard is then equal to the proportion of the essential patents that it owns. For example, if a company owns one-quarter of all the patents declared essential to a standard, then that company would be assigned one-quarter of the standard's value. The patent-counting method requires no information to implement beyond a list

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55 *See* Sendhil Mullainathan & Jann Spiess, *Machine Learning: An Applied Econometric Approach*, 31 J. Econ. Persp. 87, 92 (2017) ("The essential problem of overfitting is that we would like the prediction function to do well *out of sample*, but we only fit in-sample." (emphasis in original)); *see also* Peter Kennedy, A Guide to Econometrics 383 (MIT Press 5th ed. 2003).

56 *Data Download Tables*, PatentsView, http://www.patentsview.org/download/.

of all the patents declared essential to the standard and accurate information about the companies that own each patent.

1. *Using Patent Counting to Value Patents Declared Essential to a Standard*

The patent-counting method is straightforward to implement, because it simply requires counting all of the patents declared essential to a standard. If *x* firms have declared patents as essential to a standard, then the total number of patents declared essential to that standard is

$$N = n_1 + n_2 + \ldots + n_x = \sum^{x} i = 1, \quad (1)$$

where n_i is the number of patents declared essential by firm *i*. Each firm *i* is assigned a proportion *s* of the standard's value equal to its share of the patents,

$$s_i = \frac{n_i}{N}. \quad (2)$$

For example, if a company owns 25 patents that are declared essential to a particular standard, and if there are 100 patents in total that are declared essential to that standard, then that company's share of the standard will be equal to 25 / 100 = 0.25, or 25 percent of the value of the standard, according to the patent-counting method.

2. *Criticisms of the Patent-Counting Method*

Although the patent-counting method is simple and requires little data, it relies on the assumption that all patents are equally valuable. In his December 2017 opinion in *TCL v. Ericsson*, Judge James Selna used the patent-counting method to calculate a FRAND royalty for TCL's use of Ericsson's 2G, 3G, and 4G SEPs.[57] He acknowledged that the patent-counting method "treats every patent as possessing identical value."[58] In its appeal to the Federal Circuit, which has yet to be argued as of this writing, Ericsson has challenged as reversible error Judge Selna's reliance on patent counting, arguing that the assumption of equal patent value "contravenes fundamental principles of royalty law," such as the principle that "[r]easonable royalties must reflect 'the incremental value that the patented invention adds.'"[59] Thus, the Federal

[57] TCL Commc'n Tech. Holdings, Ltd. v. Telefonaktiebolaget LM Ericsson, No. 8:14-cv-00341, 2017 WL 6611635, slip op. at *9 (C.D. Cal. Dec. 21, 2017), *appeal docketed*, No. 18-1363 (Fed. Cir. Mar. 29, 2018).

[58] *Id.*

[59] Corrected Non-Confidential Brief for Appellants Ericsson Inc. & Telefonaktiebolaget LM Ericsson at 26, TCL Commc'n Tech. Holdings, Ltd. v. Telefonaktiebolaget LM Ericsson, No. 18-1363 (Fed. Cir. June 20, 2018) (quoting Ericsson, Inc. v. D-Link Sys., Inc., 773 F.3d 1201, 1226 (Fed. Cir. 2014)). Several *amici curiae* filed briefs with the Federal Circuit that echoed this criticism. *See* Brief of Amicus Curiae Nokia Technologies Oy in Support of Appellants Telefonaktiebolaget LM Ericsson and Ericsson Inc. at 14–15,

Circuit might soon decide whether simple patent counting is a reliable methodology for calculating a reasonable royalty for SEPs. Here, we analyze three salient economic criticisms directed at the patent-counting methodology.

a. The Empirical Evidence That Patents Have Skewed Distributions of Value

First, empirical evidence refutes the assumption that all patents have equal value. Legal and economic scholars have shown that, to the contrary, the distribution of the economic value of patents is highly skewed.[60] That is, the economic value of patents is concentrated among a handful of extremely valuable patents, and most patents contain very little economic value.

It is reasonable to expect that the distribution of economic value of SEPs is also skewed.[61] Put differently, a small number of SEPs might cover critical, high-value technologies used in implementing a standard, whereas most other SEPs might cover peripheral or ancillary technologies. In that case, the failure to account for differences in patent quality would overcompensate an SEP holder whose essential patent covers an ancillary technology and would undercompensate an SEP holder whose essential patent covers a key technology. As a result, valuing the contribution of an SEP portfolio on the basis of a simple counting exercise would reduce SEP holders' incentives to invest in

TCL Commc'ns Tech. Holdings, Ltd. v. Telefonaktiebolaget LM Ericsson (June 18, 2018) (No. 18-1363) ("A patent counting rule of thumb, if it were to be applied, should be supported to show that it adequately values the technologies at issue. Otherwise, it becomes more like the old Goldscheider [25 percent] rule of thumb—a rule of convenience without support."); Brief of InterDigital, Inc. as Amici Curiae in Support of Defendants-Appellants at 7–8, TCL Commc'ns Tech. Holdings, Ltd. v. Telefonaktiebolaget LM Ericsson (June 18, 2018) (No. 18-1363) ("[T]he district court's 'top down' methodology treats all patents as having equal value, and relies on straight 'patent counting' to determine 'shares' of SEPs attributable to each company with disclosed SEPs. Ericsson correctly notes that a 'patent counting' methodology that treats each patent as having equal value is plainly unreliable. In InterDigital's experience, patents can differ greatly in scope and in their technical contributions to a standard. And, as Ericsson further argues, in the context of patent damages law, this Court's precedent has been clear that the value added by a patented feature is a critical aspect of a reasonable royalty analysis.").

[60] *See, e.g.*, Mark Schankerman & Ariel Pakes, *Estimates of the Value of Patent Rights in European Countries During the Post-1950 Period*, 96 Econ. J. 1052 (1986); Mark A. Lemley, *Rational Ignorance at the Patent Office*, 95 Nw. U. L. Rev. 1495 (2001); John R. Allison, Mark A. Lemley, Kimberly A. Moore & Derek R. Trunkey, *Valuable Patents*, 92 Geo. L.J. 435 (2004); Bronwyn H. Hall, Adam Jaffe & Manuel Trajtenberg, *Market Value and Patent Citations*, 36 RAND J. Econ. 16 (2005); Bronwyn H. Hall, Adam B. Jaffe & Manuel Trajtenberg, *The NBER Patent Citations Data File: Lessons, Insights and Methodological Tools* 6 (National Bureau of Economic Research, Working Paper No. 8498, 2001) ("[I]t has long been known that innovations vary enormously in their technological and economic 'importance', 'significance' or 'value', and moreover, that the distribution of such 'values' is extremely skewed.").

[61] It bears emphasis that the distribution of the economic value of patents essential to a given standard is a factual inquiry requiring rigorous technical and economic analysis. Indeed, in a 1998 study, Mark Schankerman analyzed the relative value of patents in different industries and observed sharp differences in the distribution of patent value among those industries. Mark Schankerman, *How Valuable Is Patent Protection? Estimates by Technology Field*, 29 RAND J. Econ. 77, 94 tbl.5 (1998). Thus, in calculating a FRAND royalty for SEPs, the court should not put undue weight on outdated analysis that does not specifically focus on the value distribution of SEPs. Rather, courts should use recent data that are specific to SEPs for the standard at issue. *See* J. Gregory Sidak, *The Meaning of FRAND, Part I: Royalties*, 9 J. Competition L. & Econ. 931, 1019–20 (2013).

developing high-quality SEPs and increase their incentives to invest in developing low-quality SEPs, which presumably require less risk, less cost, and less time to develop than do high-quality SEPs.

By analyzing patents essential to the LRDIMM standard, we will provide evidence in line with previous research that this assumption of equal patent value is unlikely to hold.

b. Overdeclaration of SEPs as Essential to the Standard

Second, the patent-counting method is particularly susceptible to patent over-declaration.[62] SSOs typically do not analyze whether a company's declared-essential patents are in fact essential to practice the standard.[63] Thus, it is likely that the number of declared SEPs exceeds the number of patents that are truly essential for practicing the standard. Moreover, because a company may declare SEPs at its own discretion, it might overdeclare its patents as being standard-essential to exaggerate the true strength of its patent portfolio. A company might also overdeclare its patents as SEPs to reduce the risk of antitrust liability.[64] If companies declare more patents essential to a standard than are necessary for an implementer actually to practice the standard, then the patent-counting method would assign the necessary patents and the unnecessary patents equal value.

c. The Increasing Rate of Patent Grants

A third criticism of the patent-counting method is that the rate of patent grants is not constant over time; instead, it has increased in recent years.[65] This effect could cause companies with more recent patent portfolios to be weighted more highly relative to their technological contribution.

To demonstrate that the rate of patent grants has increased over time, we use our database to calculate the number of patents issued each year in

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62 *See, e.g.*, Tim Pohlmann & Knut Blind, iPlytics GmbH, Landscaping Study on Standard Essential Patents (SEPs) 8 (2016), http://ec.europa.eu/DocsRoom/documents/20741/attachments/1/translations/en/renditions/native; *see also* Jorge L. Contreras, *Essentiality and Standards-Essential Patents*, *in* Cambridge Handbook of Technical Standardization Law—Antitrust, Competition and Patent Law ch. 13, at 209, 222–25 (Jorge L. Contreras ed., Cambridge Univ. Press 2017); Robin Stitzing, Pekka Sääskilahti, Jimmy Royer & Marc Van Audenrode, Over-Declaration of Standard Essential Patents and Determinants of Essentiality (Oct. 27, 2017) (unpublished manuscript), https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2951617&download=yes; Sidak, *The Meaning of FRAND, Part I: Royalties*, *supra* note 61, at 1050.

63 *See* ETSI, Intellectual Property Rights (IPRS); Essential, or Potentially Essential, IPRs Notified to ETSI in Respect of ETSI Standards, ETSI SR 000 314 V2.23.1, at 11 (2018) ("ETSI has not checked the validity of the information, nor the relevance of the identified IPRs to the Standards or Technical Specifications and cannot confirm, or deny, that the IPRs are, in fact, essential, or potentially essential."); Sidak, *The Meaning of FRAND, Part I: Royalties*, *supra* note 61, at 957–58.

64 *See* Sidak, *The Meaning of FRAND, Part I: Royalties*, *supra* note 61, at 958–59.

65 We base this observation on our own analysis, discussed in the following pages, of data reported in *Data Query*, PatentsView, http://www.patentsview.org/query/. *See also* Michael Carley, Deepak Hegde & Alan Marco, *What Is the Probability of Receiving a U.S. Patent?*, 17 Yale J.L. & Tech. 203 (2015).

two National Bureau of Economic Research (NBER) technological categories. These categories assign patents to technology fields that cover similar technologies. (We explain the NBER technology categories in more detail in Part III.C.1.) The LRDIMM standard covers two main NBER technology categories: category 2 (computers and communications) and category 4 (electrical and electronic), and each patent is assigned to a single category. Table 1 reports the number of patents issued each year in NBER technological categories 2 and 4.

Table 1. Number of Patents Granted Annually in NBER Technology Categories 2 and 4 (1976–2014)

Year	Number of Patents Granted in NBER Technology Category 2	Number of Patents Granted in NBER Technology Category 4
1976	4,674	11,199
1977	4,672	10,758
1978	4,744	10,441
1979	3,628	7,838
1980	4,723	9,553
1981	4,648	10,261
1982	4,960	10,020
1983	4,749	9,709
1984	5,498	11,749
1985	6,380	12,586
1986	6,737	12,396
1987	8,798	16,027
1988	8,605	14,192
1989	11,326	17,219
1990	10,231	16,221
1991	10,886	17,635
1992	11,306	17,865
1993	12,760	17,492
1994	14,608	18,875
1995	15,826	19,464
1996	19,228	20,639
1997	19,804	20,280
1998	30,762	27,153
1999	31,505	28,632
2000	32,543	31,875

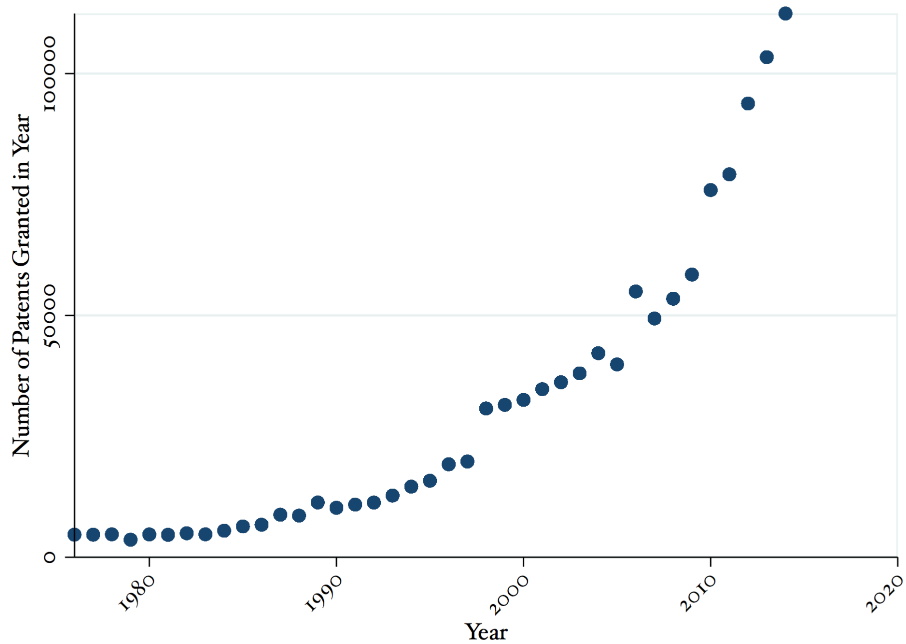
Year	Number of Patents Granted in NBER Technology Category 2	Number of Patents Granted in NBER Technology Category 4
2001	34,755	35,301
2002	36,194	37,310
2003	38,034	38,495
2004	42,187	39,355
2005	39,882	34,822
2006	54,961	40,852
2007	49,363	37,991
2008	53,467	38,349
2009	58,457	41,260
2010	75,926	51,314
2011	79,185	50,996
2012	93,816	54,490
2013	103,407	59,206
2014	112,424	65,006
Total	1,165,659	1,024,826

Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Figure 2 shows the number of patents issued each year in NBER technological category 2, and Figure 3 shows the number of patents issued each year in NBER technological category 4. Both categories show an increasing number of patents issued in each year over time.[66]

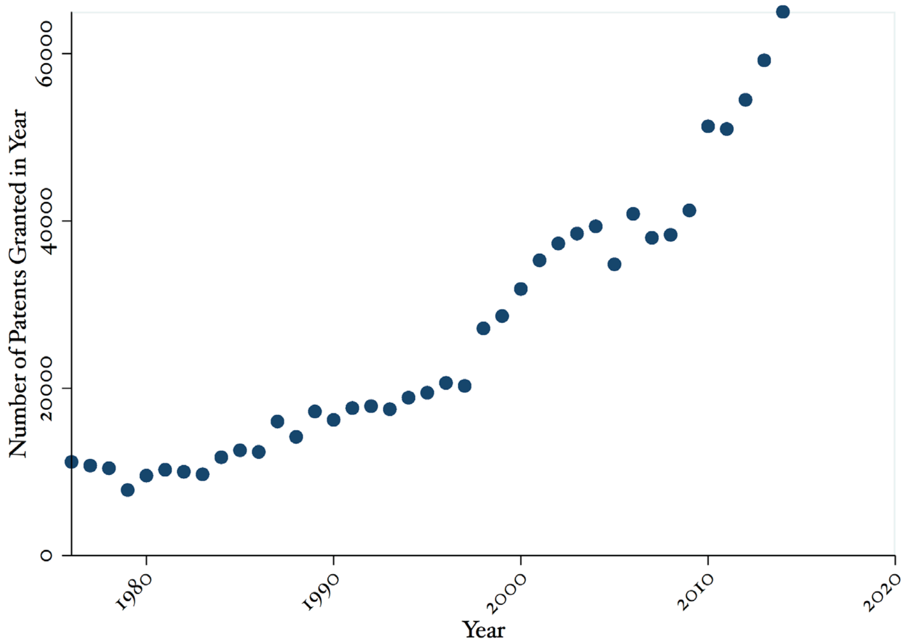
[66] We exclude from our analysis data from years after 2014 because only partial information is available for those years.

Figure 2. Number of Patents Issued Annually
in NBER Technology Category 2 (1976–2014)



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.
Note: Patents issued numbered 4,674 in 1976 and 112,424 in 2014 in NBER technology category 2. The median patent was issued in 2007. Only partial information was available for 2015 and 2016, so we exclude those years. No information was available for 2017 when we conducted the research for this article.

Figure 3. Number of Patents Issued Annually in NBER Technology Category 4 (1976–2014)



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.
Note: Patents issued numbered 11,199 in 1976 and 65,006 in 2014 in NBER technology category 4. The median patent was issued in 2007. Only partial information was available for 2015 and 2016, so we exclude those years. No information was available for 2017 when we conducted the research for this article.

Figures 2 and 3 show that the Patent Office has issued patents in NBER technology categories 2 and 4 at increasing rates over time. This evidence shows that relying on a simple patent-counting method could overstate the value of more recent patent portfolios relative to their technological contribution, especially if the increase in patent grant rates is due to some reason other than increased innovation.

B. *Forward-Citation Analysis*

One improvement over the patent-counting method has been to use forward citations to weight the value of patents, and then to apportion a firm's contribution to the value of a standard by the weighted sum of the value of the firm's patents.

A forward citation is a citation from a patent issued after the original patent that references the original patent as prior art.[67] That is, a count of forward citations is a measure of the number of future patents that are, in part, related to the original patent. Forward citations have frequently been used as a measure of a patent's value, on the economic rationale that more valuable patents inspire more research and future patentable inventions.[68] Manuel Trajtenberg found that citation-weighted patent counts were more closely correlated with R&D output than were unweighted patent counts.[69] The number of forward citations varies greatly among patents, as some patents receive many citations, whereas most patents receive relatively few citations.

Several studies have examined the relationship between a patent's number of forward citations and that patent's private value to the patent owner. David Abrams, Ufuk Akcigit, and Jillian Popadak found an inverted-U relationship in which the value of a patent increases initially with the number of citations to that patent but decreases beyond a certain number of citations.[70] Such a relationship between patent value and patent citations implies that the least valuable patents and most valuable patents receive fewer citations than do patents with an average value. The authors inferred from their results that this relationship results from strategic patenting in which a firm applies for patents to protect its initial technological investment and to reduce the ability of other companies and inventors to obtain patents in the same field, the side-effect of which, ironically, is to reduce the number of citations by third parties to those "strategic" patents. However, it is worth noting that these authors relied on a private dataset of patents that was purchased by a non-practicing entity. Thus, those authors' results might be affected by sample selection, where companies were unwilling to sell their most valuable patents.

In contrast, in her doctoral dissertation, Kirti Gupta, now the Senior Director of Economic Strategy at Qualcomm, relied on internally developed patents that are still owned by their original assignee whose economic value was rated by subject-matter experts. A patent's value, for Gupta's analysis, was "economic value determined by the likelihood of products infringing on the

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[67] *See* USPTO, Manual of Patent Examining Procedure § 2121 (2018), https://www.uspto.gov/web/offices/pac/mpep/s2121.html.

[68] Adam B. Jaffe & Gaétan de Rassenfosse, *Patent Citation Data in Social Science Research: Overview and Best Practices*, 68 J. Ass'n Info. Sci. & Tech. 1360, 1361 (2017); *see also* Nils Omland, *Valuing Patents Through Indicators*, *in* The Economic Valuation of Patents 169, 174 (Federico Munari & Raffaele Oriani eds., Edward Elgar 2011).

[69] Manuel Trajtenberg, *A Penny for Your Quotes: Patent Citations and the Value of Innovations*, 21 RAND J. Econ. 172 (1990).

[70] David S. Abrams, Ufuk Akcigit & Jillian Popadak, *Patent Value and Citations: Creative Destruction or Strategic Disruption?* (National Bureau of Economic Research, Working Paper No. 19,647, 2013).

patent, and the size of the product market."[71] She then examined those valuations in licensing negotiations within the Information and Communication Technology sector. Gupta found that the number of forward citations that a patent receives is the "single biggest factor in predicting the economic value of patents."[72] She explained that "[o]ther factors, such as citations made by a patent (a.k.a. backward citations), the number of claims, family size, and office actions, do not carry significant predictive power."[73]

1. *Inequality in the Distribution of Forward Citations Across Patents Classified in NBER Technology Categories 2 and 4*

Consistent with the existing literature, we find that the distribution of forward citations across patents classified in NBER technology categories 2 and 4 is unequal. Table 2 shows summary statistics of forward-citation counts for NBER technology categories 2 and 4. The results show the long-tail of the citation distribution. For NBER technology category 2, the median number of citations is four, meaning that half of the patents have four or fewer citations and half of the patents have more than four citations. Three-quarters of the patents have 15 or fewer citations, and 99 percent of the patents have 174 or fewer citations. The four patents with the most forward citations have citation counts of 1878, 1897, 2378, and 2465. These results indicate that a small number of patents receive a large number of citations, whereas most patents receive relatively few citations.

Table 2. Summary Statistics on the Number of Forward Citations to Patents Classified in NBER Technology Category 2 or 4 for Patents Granted from 1976 Through 2014

NBER Category	Number of Patents	Mean Citation Count	50th Percentile	75th Percentile	90th Percentile	95th Percentile	99th Percentile
2	1,165,659	16.49	4	15	41	71	174
4	1,024,826	12.16	5	13	29	46	112

Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

To demonstrate graphically the inequality in the distribution of patent citations, we also plot Lorenz curves, which economists often use to analyze

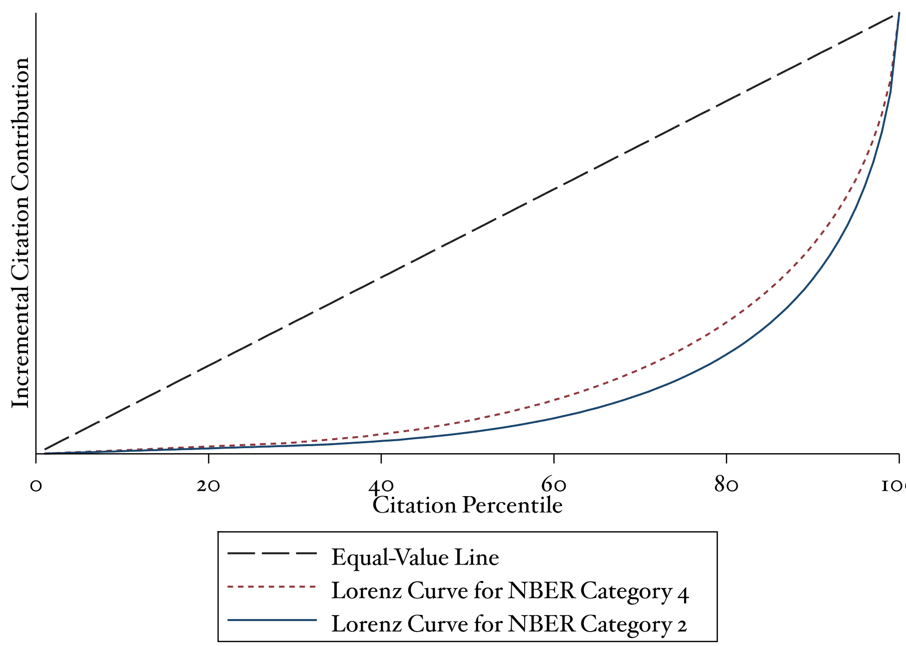
[71] Kirti Gupta, Economics of Intellectual Property: Valuation, Strategy and Policy Impact 9 (2012) (unpublished Ph.D. dissertation, University of California, San Diego), https://cloudfront.escholarship.org/dist/prd/content/qt3hn290cs/qt3hn290cs.pdf?t=msz1r3.

[72] *Id.* at 8.

[73] *Id.*

the degree of inequality in the distribution of data.[74] Figure 4 shows the Lorenz curves for the distribution of patent citations for patents in NBER technology categories 2 and 4. To plot the Lorenz curves, we first rank the patents in each NBER technology category by the number of forward citations and categorize those patents into citation percentiles. We then calculate the total number of forward citations for patents in each percentile. Finally, we calculate the relative incremental citation contribution from each percentile by dividing the total number of forward citations in that percentile by the total number of forward citations in the NBER technology category. The *y*-axis shows the relative incremental citation score contribution and the *x*-axis shows the citation percentile.

Figure 4. Lorenz Curves for Patents Classified in NBER Technology Category 2 or 4 for Patents Granted from 1976 Through 2014



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Using the Lorenz curves in Figure 4, we calculate the Gini coefficients for each distribution of forward citations to estimate the degree of inequality in the distribution of patent value in NBER technology categories 2 and 4. The Gini coefficient represents the ratio of the area between the Lorenz curve

[74] *See, e.g.*, Mary Jean Bowman, *A Graphical Analysis of Personal Income Distribution in the United States*, 35 Am. Econ. Rev. 607, 617 (1945); Max O. Lorenz, *Methods of Measuring the Concentration of Wealth*, 9 Publications Am. Stat. Ass'n 209 (1905).

and the 45-degree line to the entire area under the 45-degree line[75] and is "[t]he most common measure of inequality."[76] In a distribution in which each patent has an equal citation score, each patent would lie on the 45-degree line (or equal-value line), and the Lorenz curve would have a Gini coefficient of zero.[77] In contrast, an NBER technology category whose value is attributable solely to one patent—a distribution that is "perfectly unequal"—would have a Gini coefficient of one.[78] Thus, a higher Gini coefficient indicates higher inequality within a distribution.

We find that the Lorenz curve for patents in NBER technology category 2 has a Gini coefficient of 0.7340, whereas the Lorenz curve for patents in NBER technology category 4 has a Gini coefficient of 0.6691. Those Gini coefficients indicate that the distribution of forward citations for patents in NBER technology category 2 is less equal than the distribution of forward citations for patents in NBER technology category 4. In addition, these statistical results confirm that patents do not have equal influence as prior art. Nor can one reliably assume that the patents in these two NBER technology categories have equal value.

2. *Trends in Citation Counts Over Time for Patents Classified in NBER Technology Categories 2 and 4*

Just as the number of patents granted has varied over time, so also has the number of citations that patents receive varied over time. Table 3 reports the number of citations received by patents granted in each year in NBER technological categories 2 and 4.

[75] Corrado Gini, *Measurement of Inequality of Incomes*, 31 Econ. J. 124, 125 (1921). We describe different methodologies that economists use to calculate the Gini coefficient in Sidak & Skog, *Hedonic Prices and Patent Royalties*, *supra* note 4, at 677–80.

[76] Joseph L. Gastwirth, *The Estimation of the Lorenz Curve and Gini Index*, 54 Rev. Econ. & Stat. 306, 307 (1972); *see also* James Morgan, *The Anatomy of Income Distribution*, 44 Rev. Econ. & Stat. 270, 270 (1962) ("It generally has been agreed, after much discussion, that the best single measure of inequality is the proportion of the triangular area on a Lorenz diagram which falls between the Lorenz curve and the diagonal, often called the Gini Index of concentration.").

[77] *See* Morgan, *supra* note 76, app., at 281 ("Clearly, perfect equality would result in points along the 45° line, and if one person had all the income the bottom and right straight lines would result. Actual curves fall in between, and the closer to the diagonal, the less the inequality.").

[78] *See id.*

Table 3. Number of Citations Received by Patents Granted Annually in NBER Technology Categories 2 and 4 (1976–2014)

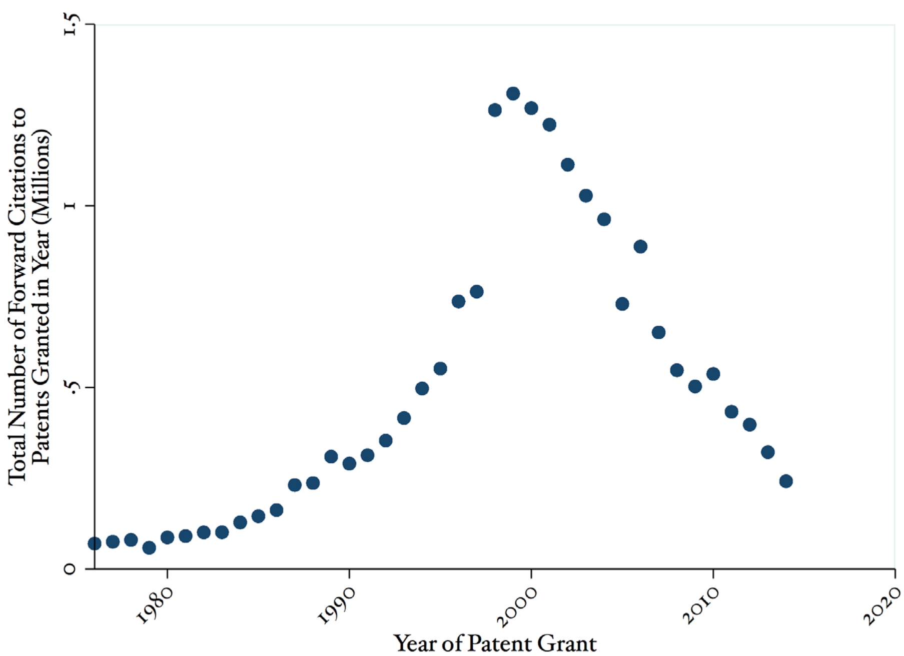
Year	Number of Citations Received by Patents Granted in NBER Technology Category 2	Number of Citations Received by Patents Granted in NBER Technology Category 2
1976	69,961	135,165
1977	75,189	137,266
1978	80,124	135,674
1979	58,460	104,082
1980	86,915	129,742
1981	90,725	146,740
1982	100,779	144,267
1983	101,195	147,744
1984	128,319	175,751
1985	145,528	197,513
1986	162,097	196,976
1987	231,229	271,446
1988	236,736	249,456
1989	309,433	309,922
1990	290,167	297,203
1991	313,319	325,853
1992	353,682	340,958
1993	415,634	357,265
1994	497,011	389,083
1995	551,880	417,481
1996	736,688	454,831
1997	763,595	468,065
1998	1,263,517	602,846
1999	1,309,139	619,027
2000	1,268,821	669,831
2001	1,223,497	696,503
2002	1,113,297	639,936
2003	1,027,917	598,091
2004	962,897	512,186
2005	729,674	404,989
2006	887,895	415,710
2007	651,372	329,864
2008	547,224	289,728

Year	Number of Citations Received by Patents Granted in NBER Technology Category 2	Number of Citations Received by Patents Granted in NBER Technology Category 2
2009	502,752	245,519
2010	536,942	255,807
2011	432,983	210,711
2012	397,523	180,343
2013	321,704	143,952
2014	241,600	113,696
Total	19,217,420	12,461,222

Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

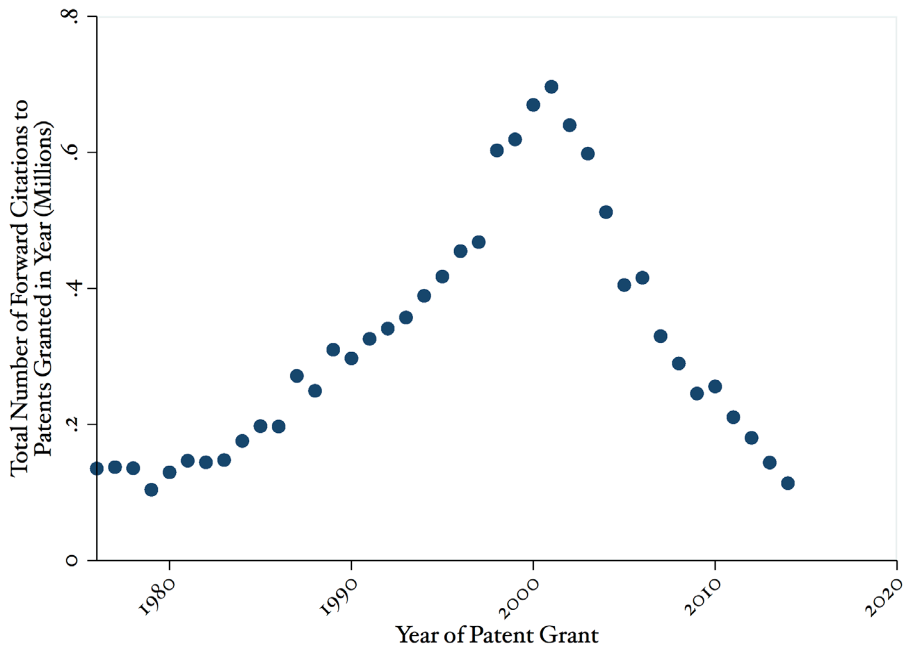
In Figures 5 and 6, we illustrate this forward-citation relationship over time. Each figure shows the number of citations issued to patents granted in a given year for a particular NBER technology category.

Figure 5. Number of Citations to Patents Granted in a Given Year for NBER Technology Category 2



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Figure 6. Number of Citations to Patents Granted in a Given Year for NBER Technology Category 4

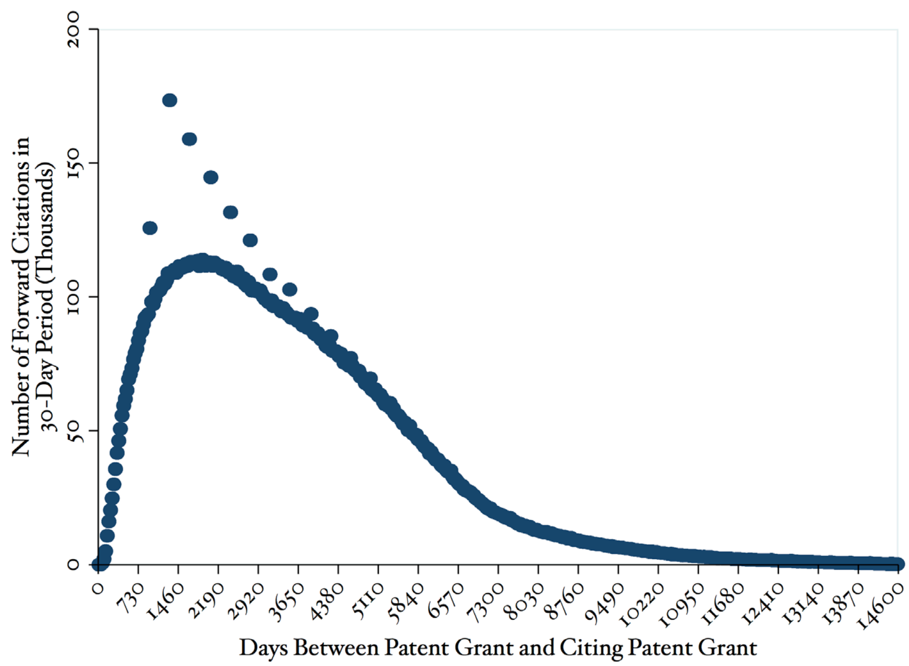


Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Figures 5 and 6 show that both of the NBER technology fields in question exhibit similar patterns. The underlying figures, which we report in Table 3, show the total number of citations to patents granted each year. Table 3 shows, for example, that, for patents in NBER technology category 2, there were 290,167 forward citations to patents granted in 1990, whereas there were 1,309,139 citations to patents granted in 1999. The number of citations to patents granted in each year increased over time before peaking at 1,309,139 in 1999 for patents in NBER technology category 2 and at 696,503 in 2001 for patents in NBER technology category 4. After those peak years, the numbers of citations to patents granted in each year declined.

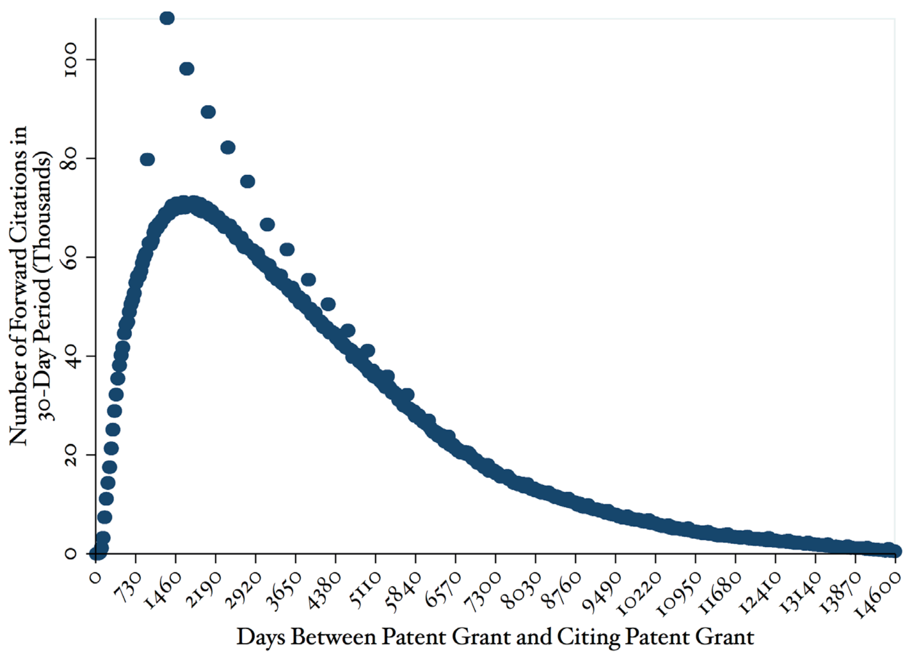
The dynamics of patent citations to prior art might explain the pattern of citations in Figures 5 and 6. For older patents, more time has passed during which new patents that could cite the older patents as prior art could be issued, increasing the number of new patents that could potentially cite to each older patent as a prior art. However, beyond a certain age, the likelihood that a patent will be cited by each new patent declines as technology advances. This trend can lead to an increasing pattern in the number of citations that halts after some time, while more recent patents continue to accumulate citations until they, too, see the number of new citations slow and potentially cease. In other words, patents are unlikely to be cited quickly; but patents also eventually might become outmoded, such that recent patents no longer cite to them. We observe that the data are consistent with this behavior in Figures 7 and 8, which show the length of time between when a patent is issued and when a patent that cites it is issued. That is, these graphs show the "age" of the patent when it receives a forward citation. We divide the "citation ages" into 30-day intervals and graph the distributions over time.

Figure 7. Citation Ages for Patents in NBER Category 2



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Figure 8. Citation Ages for Patents in NBER Category 4



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Both NBER technology categories exhibit similar patterns. The number of citations increases quickly as patents age; it peaks when patents are approximately four years old. The number of new patents that cite each original patent then decreases over time.

There is a second pattern wherein some months exhibit significantly higher numbers of citations than do the surrounding months. We believe that this phenomenon is due to data cleaning. Some patents had nonsensical or uncertain citation dates. For those patents, we assigned citation dates at the beginning of each year.

Because of these time and patent-age issues in the numbers of citations a patent has received, researchers might choose to adjust their citation measures for when a patent was granted or for the patent's age.

3. *Estimating a Patent's Relative Value Using Forward-Citation Weighting*

One can use a forward-citation adjustment to calculate the relative worth of each patent in the standard by adjusting each patent's value according to some weight. That weight is calculated using various formulas that have been adapted to meet the available data and the needs of the researchers. For a given standard, the total weighted value is calculated as shown in Equation 3:

$$s_i = \sum_i w_i. \tag{3}$$

Each patent *i* has an unweighted value of one. A patent is then assigned an individual weight w_i according to some function of the patent's citations. These weighted values are then summed to determine the total value of the standard, *s*. The share of the value assigned to each patent is then its weighted value divided by the total value of the standard, as shown in Equation 4:

$$s_i = \frac{w_i}{s}. \tag{4}$$

The simplest measure of forward citations is simply an unweighted count of all future patents that cite back to the original patent. For this particular citation-weighting methodology, the weight applied to each patent is the number of future citing patents. However, various factors can influence the number of citations that a patent receives for which a researcher might want to adjust, including the timing of the citations, the patent's technology field, and the patent's age.

We refine our measure of weighting each patent by the number of forward citations that it has received to control for how the age of the patent might have affected the number of citations that it has received. One simple method is to omit all citations that occur after a set length of time. We perform that adjustment in two ways. First, we include only citations that occurred within ten years of a patent's grant date. Second, we include only citations that occurred within four years of a patent's grant date. Once we have eliminated the "older" citations, we sum the number of forward citations that each patent received and use those weights to apportion the value of the standard to its constituent patents.

For patents essential to the LRDIMM standard, the median interval between a patent's grant date and its citation date was 4.81 years, whereas the average interval between a patent's grant date and its citation date was 5.05 years. Thus, a shorter interval might indicate faster-than-average citations, whereas a longer interval might indicate a longer period of time spent developing the standard after the declaration of the first patent.

4. *Using Forward-Citation Analysis to Value Patents Declared Essential to the LRDIMM Standard*

We calculate the citation-weighted score for each patent in the LRDIMM standard using each of the three measures mentioned above. First, we use an unadjusted count of the number of forward citations that each patent has received. Second, we control for citation age by omitting all citations that occur more than ten years after the patent was granted. Third, we apply a stricter control for a citation's age by omitting all citations that occur more than four years after the original patent was granted.

For a simple count of citations, each patent receives the weight shown in Equation 5:

$$w_i = \sum_c I_c, \tag{5}$$

where w_i is the weight applied to patent i, c denotes all of the patents that cite patent i as prior art, and I_c is an indicator that takes a value of one if patent c cites patent i and is zero otherwise. That is, each patent's weight is simply a count of the total number of later patents that cite it as prior art. This weight can be used in Equation 6 to calculate each patent's weighted score, the total score of the standard, and each company's share of the standard's value.

We can similarly calculate the weighted values of each patent with the ten-year and four-year cutoffs applied, where we count only citations made within a specified interval of time after the grant of the initial patent. This formula is shown in Equation 6:

$$w_i = \begin{cases} \sum_c I_c, \Upsilon_c - \Upsilon_i \leq K; \\ \sum_c 0, \Upsilon_c - \Upsilon_i > K. \end{cases} \tag{6}$$

The total weight for each patent is the sum of two separate weighting functions. Within the time limit—when the citing patent's year of issue minus the base patent's year of issue is less than the year cutoff value, or when $\Upsilon_c - \Upsilon_i < K$—all citations are counted equally and a patent's weight is equal to the number of patents that listed it as prior art. Outside of this window, all citing patents are assigned a weight of zero and do not increase the base patent's citation score. After these weights are calculated for each of the patents, we can calculate the total score for the standard, as well as each SEP's share of the value of the standard. In our analysis, we assign to K a cutoff value of either four or ten years, depending on the degree to which we want to value a citation's recency.

C. *Forward Citations Relative to a Patent's Field or Cohort*

Just as the number of citations that a patent receives might vary with time and age, so might the number of citations vary according to the patent's subject matter. That is, more patents might be granted or patents might have more citations in certain technology fields, which would result in a greater number of forward citations and, therefore, a higher weighting in the apportionment calculation.

In the context of SEPs, controlling for a patent's technology field (for example, as defined by the NBER technology categories) might be unnecessary because the patents have already been declared essential to the same technological standard. That is, patents declared essential to the same standard might be more technologically related to each other than are other patents that are not declared essential to the same standard but belong in the same NBER technology subcategory. In other words, the standard itself might serve as a more precise technological category than an NBER technology category.[79]

However, some standards, such as certain wireless standards, might cover patents in a variety of fields and concern diverse devices, such as handsets and base stations. In those cases, an expert might conclude that it is useful to make a technological adjustment appropriate for the standard—for example, by separating patents on the basis of NBER technology categories. Although the LRDIMM standard itself might be the more useful technological grouping, for purposes of general exposition we examine here the effect of controlling for NBER technological categories on patent grant rates and citation likelihood.

1. *The NBER Patent Classification System*

The NBER patent classification system was designed by Bronwyn Hall, Adam Jaffe, and Manuel Trajtenberg in the course of a decades-long undertaking to build a comprehensive database of U.S. patents.[80] The NBER Patent Data Project aimed to improve the accessibility of U.S. patent data for researchers, and to address some of the failings of earlier efforts to structure the data.[81] When the database was published in 2001, it included 2,923,922 utility patents granted between January 1963 and December 1999 and approximately 16 million citations made to those patents between 1975 and 1999.[82] Subsequently, the data have been supplemented several times through 2006.[83] The database that we use for our analysis assigns NBER categories to patents, including more recent patents, using the same method and consists of 5,105,937 patents granted through May 2015.[84]

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[79] In Appendix I, we provide examples of patents in NBER technology category 2 that belong in different NBER technology subcategories to illustrate the range of technologies that an NBER technology category might contain.

[80] Hall, Jaffe & Trajtenberg, *The NBER Patent Citations Data File: Lessons, Insights and Methodological Tools*, *supra* note 60, at 3, 7.

[81] For example, a researcher might need to obtain information on the citations *received* by a given patent, as opposed to the citations *made* by that patent. However, switching from backward citations to searching for forward citations often requires cumbersome restructuring of the original data. *Id.* at 7.

[82] Hall, Jaffe, and Trajtenberg used the 1999 USPTO classification. *Id.* at 12 n.13.

[83] *Patent Data Project*, National Bureau of Economic Research, https://sites.google.com/site/patentdataproject/Home; *see also* Jean Roth, *The NBER U.S. Patent Citations Data File: Lessons, Insights, and Methodological Tools*, National Bureau of Economic Research, http://www.nber.org/patents/.

[84] *Data Download Tables—NBER*, PatentsView, http://www.patentsview.org/download/.

Hall, Jaffe, and Trajtenberg understood the practical limitations of the USPTO's elaborate patent-classification system, which at the time consisted of 417 main patent classes and approximately 120,000 patent subclasses.[85] As they noted in a 2001 working paper, "even 400 classes [were] far too many for most applications (such as serving as controls in regressions)."[86] Furthermore, a classification system that is designed to be useful for understanding the economic relationships between technologies or the process of innovation has a different purpose than one designed to meet the objectives of a patent office. To improve the utility of the data for researchers, Hall, Jaffe, and Trajtenberg organized the patents in their database into 6 main technological categories and 36 subcategories.[87] However, since 2001, 1 main category and 2 subcategories have been added, such that there are now 7 main categories and 38 subcategories.[88] These NBER categories represent groups of various "Cooperative Patent Classification" codes that are used by both the USPTO and the European Patent Office for classifying patents.[89] Thus, the NBER codes represent a reorganization of different low-level patent groupings, rather than a new grouping on the basis of a patent's innovative technology.

The Patent Data Project actively maintains and updates the U.S. patent database and NBER's patent-classification system. All data, including information on patent citations and assignees, are freely available through the Project's website, which is supported by the NBER.

2. *NBER Classification of Patents Declared Essential to the LRDIMM Standard*

The PatentsView database provides the NBER technology category and subcategory assignment for each patent. We examine the NBER technology category and subcategory assigned to each patent declared essential to the LRDIMM standard. After producing that list, we find that the LRDIMM standard is composed of patents in two major technology categories and twelve subcategories. Table 4 shows these NBER categories and subcategories.

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85 Hall, Jaffe & Trajtenberg, *The NBER Patent Citations Data File: Lessons, Insights and Methodological Tools, supra* note 60, at 12.

86 *Id.*

87 *Id.* at 12–13.

88 *Data Download Tables—NBER*, PatentsView, http://www.patentsview.org/download/.

89 Press Release, USPTO, USPTO and EPO Announce Launch of Cooperative Patent Classification System (Jan. 2, 2013), https://www.uspto.gov/about-us/news-updates/uspto-and-epo-announce-launch-cooperative-patent-classification-system.

Table 4. NBER Technology Categories and Subcategories for Patents Declared Essential to the LRDIMM Standard

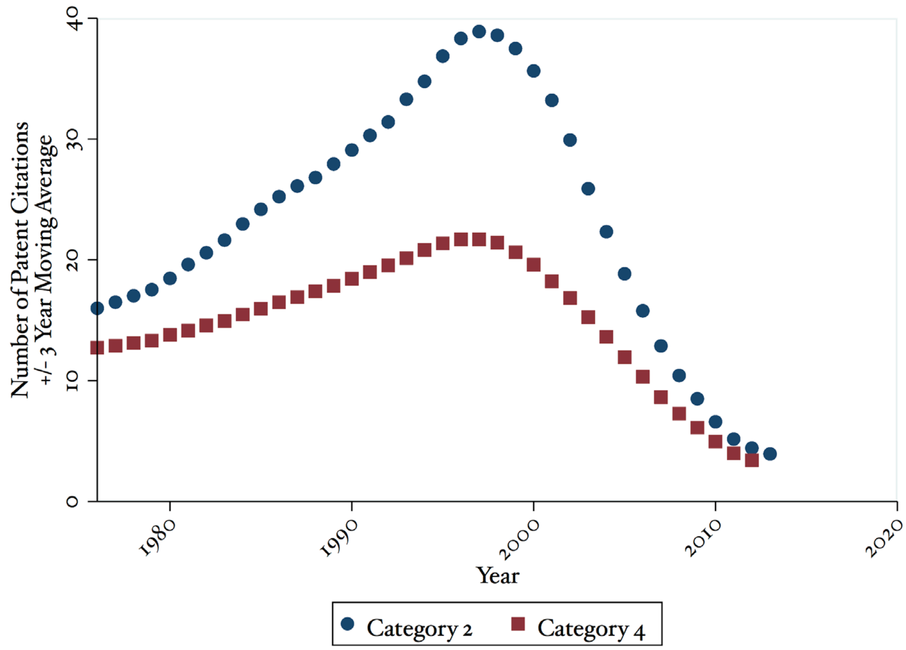
	ID	Title	ID	Title
Categories	2	Computers & Communications	4	Electrical & Electronic
Subcategories	21	Communications	41	Electrical Devices
	22	Computer Hardware & Software	42	Electrical Lighting
	23	Computer Peripherals	43	Measuring & Testing
	24	Information Storage	44	Nuclear & X-rays
	25	Electronic business methods and software	45	Power Systems
			46	Semiconductor Devices
			49	Miscellaneous

Source: *Data Download Tables—NBER*, PatentsView, http://www.patentsview.org/download/.

Under NBER's classification system, each patent belongs to only one category and one subcategory. It is possible to weight a patent's valuation by comparing its citations to the citations of other patents in the same category or subcategory. We find that patents declared essential to the LRDIMM standard are categorized under two different patent technology categories and 12 different patent technology subcategories.

In Figure 9 we show the average number of citations to patents in NBER technology categories 2 and 4 over time. The averages are calculated over a rolling six-year window, as we explain further below.

Figure 9. Average Citations to Patents in NBER Technology Categories 2 and 4 by Year of Patent Grant

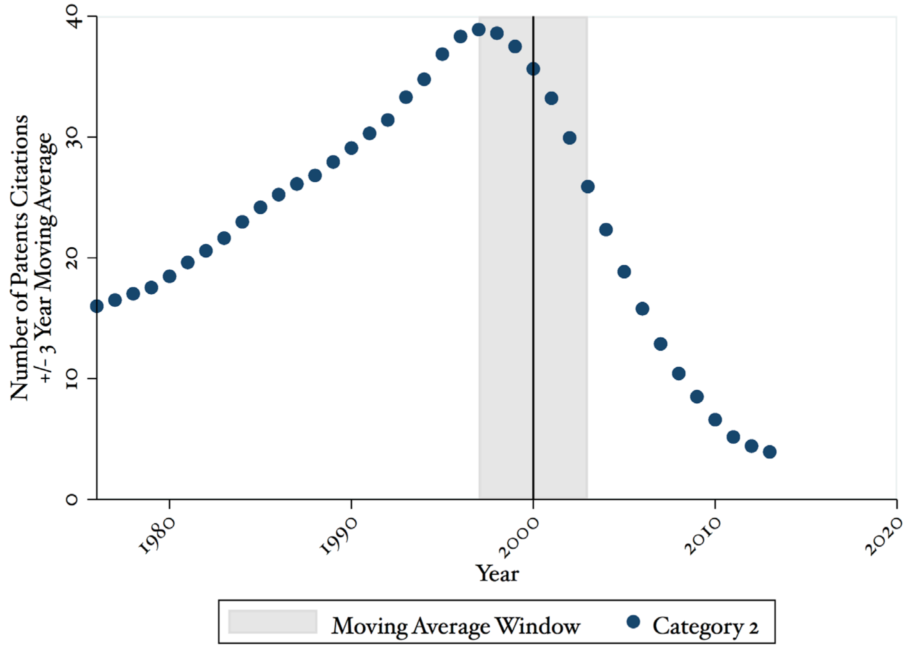


Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Figure 9 shows that, on average, patents in category 2 have more citations than do those in category 4. However, both categories follow similar trends whereby the number of citations increases until approximately the late 1990s, before declining for more recently granted patents.

To calculate the averages shown in Figure 9, we used a rolling six-year window. This window covers the period three years before and three years after any given year. We then averaged the total number of citations received by all patents issued in any of those years that were in the relevant technology category. For example, to calculate the value for NBER category 4 in 2000, we calculated the total number of citations received by all patents granted between 1997 and 2003 and then averaged that value across all of the patents granted in those years. The citing patents themselves could be granted in any year. This time window for 2000, as a representative year, is shown in Figure 10. Although we use the year 2000 as a representative example, we repeat this calculation for all years to calculate the average number of citations to patents granted in each year.

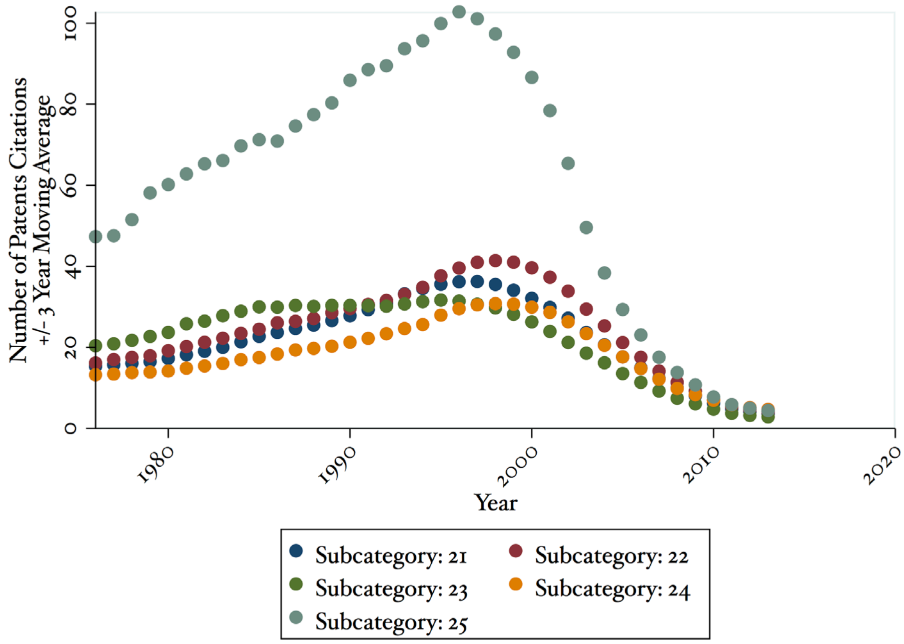
Figure 10. The Time Window for Calculating the Average Number of Citations to Patents in a Time-Technology Cohort



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

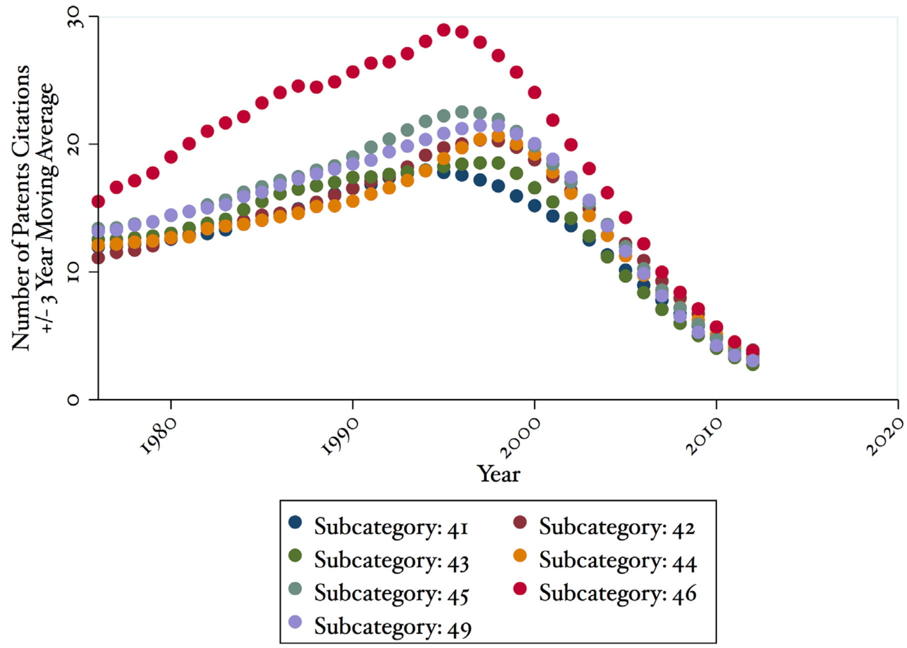
In addition to being assigned to broad NBER technology categories, patents are also assigned to an NBER technology subcategory. We repeat the averaging procedure but define the time-technology cohorts as patents in a particular NBER technology subcategory granted within three years of the base year. Figure 11 shows the trends for the subcategories of NBER technology category 2, and Figure 12 shows the trends for the subcategories of NBER technology category 4.

Figure 11. Average Citations to Patents in NBER Technology Subcategories of Category 2 by Year of Patent Grant



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Figure 12. Average Citations to Patents in NBER Technology Subcategories of Category 4 by Year of Patent Grant



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

Figures 11 and 12 show the trends in the number of citations received by patents in a particular time-technology cohort. The broad trends are similar over time, but some subcategories have more citations than others. Patents in subcategory 25 of NBER technology category 2 (electronic business methods and software) in particular have more citations on average than do patents in the other subcategories. Within the subcategories of NBER technology field 4, patents in subcategory 46 (semiconductor devices) have continued to have more citations, on average, than have patents in other subcategories.

3. *Methodology and Criticisms*

A researcher might find it useful to control for these variations across and within NBER technology categories. For example, we divide each patent's citation count by the average number of citations received by other patents in its time-technology cohort to find its citation value relative to comparable time-technology patents. We then sum these ratios for all LRDIMM SEPs and calculate each patent's share of the total. We use this value to determine each patent's share of the standard's value.

This adjustment might be more necessary in some circumstances than in others. For example, a portfolio of patents from many different technology fields might be more subject to differing citation patterns across fields than would be a portfolio of technologically similar patents, where the fields have similar citation practices. In the case of SEPs, patents that are essential to practicing a similar technology might not require this adjustment. In the case of the LRDIMM standard in particular, we find that the SEPs are assigned to two broad technology categories and twelve subcategories. Because those patents all relate to the same standardized technology, a researcher would need to decide whether it is appropriate to compare patterns of forward citation to patents reading on the same standard to patterns of forward citation to patents in different technology groups that do not read on that standard.

There are also other methods of patent classification (such as the Cooperative Patent Classification codes, from which the NBER technology categories are derived), and those patent classifications might have been developed to serve purposes other than to compare the relative value of patents. Perhaps the patents were assigned to cohorts that confound the researcher's objective. A researcher should consider the role and effect of these category assignments to determine whether the adjustment that we explain here is appropriate for a given forward-citation analysis.

D. Age Adjustments

In our 2017 article,[90] we proposed a method of age adjustment that attempts to control for a patent's age as well as the speed with which forward citations occur. We assign each citation a separate weight (an adjustment that we did not make in any of the methods previously discussed here). We weight each citation according to the amount of time that has elapsed between the grant of the patent and the forward citation to that patent.

According to this scoring technique, forward citations that occur soon after a patent is granted receive more weight than a forward citation that occurs long after the patent is granted. This weighting technique adjusts for the age of citations, but it does not discard any citations. That is, a forward citation that occurs 11 years after the grant of the patent still counts in our measure, as it still provides useful information that the patent has served as prior art for a later patent. In contrast, the time-censored methods of citation weighting that we discussed above would not account for this kind of late citation.[91]

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[90] Sidak & Skog, *Hedonic Prices and Patent Royalties*, *supra* note 4.

[91] Consider, as an analogy to forward citation in economics, the classic article on Ramsey pricing. *See* Frank P. Ramsey, *A Contribution to the Theory of Taxation*, 37 ECON. J. 47 (1927). Perhaps because his untimely death cut short a promising career, Ramsey's insights gained little attention for nearly three decades. *See* Marcel Boiteux, *Sur la Gestion des Monopoles Publics Astreints à l'Équilibre Budgetaire*, 24 ECONOMETRICA 22

Equation 7 expresses our age-adjustment for an individual patent:

$$\textit{Patent-Citation Score} = \sum_{Citations} (0.5)^{t/h}, \quad (7)$$

where *t* is the number of days between the date of each cited patent's issuance and the date of the citing patent's issuance, and *h* is the half-life of a patent citation. Merriam-Webster defines half-life as "the time required for half of something to undergo a process."[92] For example, in nuclear physics, half-life measures "how long it takes for half the nuclei of a piece of radioactive material to decay."[93] Similarly, in environmental science, half-life measures the length of time needed for half the amount of a radioactive tracer to be eliminated from an ecosystem.[94] For purposes of our analysis, we define half-life as the length of time it takes for a citation's value to decrease to half of its original value. In other words, the half-life of a patent citation measures the rate at which a citation's value decays, with a shorter half-life indicating a higher rate of decay in citation value (and consequently, more relative value to more recent citations). For purposes of patent citation weighting, the choice of a particular half-life value is a judgment made by the technological expert. For the half-life, we use the median number of days between the grant of the original patent and the grant of the forward-citing patent for all patents analyzed. Using the median number of days until a citation as the half-life means that a citation that occurs at the median time will have a score of 0.5. The half of citations that occur closer to the patent grant date will have scores closer to one, and the other half of citations, which are made a longer time after the grant date, will have scores closer to zero.

For example, suppose that patent *A* receives three citations—one citation from a patent issued 10 days after the issuance of patent *A*, one citation from a patent issued 20 days after the issuance of patent *A*, and one citation from a patent issued 30 days after the issuance of patent *A*. For this hypothetical example, the median length of time between patent issuance and citing-patent issuance is set at 20 days (which is the half-life we use in this calculation). The citation from the patent issued 10 days after the initial grant receives a score above 0.5, the citation at 20 days—the median value—receives a

(1956). More than 40 years passed by the time Ramsey pricing was "rediscovered" by American economists in William J. Baumol & David F. Bradford, *Optimal Departures from Marginal Cost Pricing*, 60 Am. Econ. Rev. 265 (1970), and its ramifications for regulatory economics widely understood. Google Scholar reports only one citation to Ramsey's article between 1927 and 1956, ten citations between 1927 and 1970, but approximately 3,300 citations between 1970 and 2018.

92 *Definition of Half-Life*, Merriam-Webster, https://www.merriam-webster.com/dictionary/half-life.

93 *Half-Life*, BBC, http://www.bbc.co.uk/schools/gcsebitesize/science/aqa_pre_2011/radiation/radioactiverev7.shtml; *see also* Robert Eisberg & Robert Resnick, Quantum Physics 559 (John Wiley & Sons 2d ed. 1985) ("Further indicated is the ***half-life*** $T_{1/2}$, which is the time required for the number of undecayed nuclei to decrease by a factor of 2.").

94 *See Definition of Half-Life*, Merriam-Webster, *supra* note 92.

score of 0.5, and the citation at 30 days receives a score below 0.5. The total patent-citation score for patent *A* is the sum of these three individual scores:

$$0.5^{(10/20)} + 0.5^{(20/20)} + 0.5^{(30/20)} = 1.56. \quad (8)$$

This valuation method examines only the time between a patent's grant and the issuance of follow-on technology.

Various disciplines have used exponential weighting of forward citations over time. For example, Erjia Yan and Ying Ding applied an exponential weight to account for the speed with which journal articles are cited.[95] In the specific context of patent citations, Hall, Jaffe, and Trajtenberg employ a quasi-structural model based on earlier research.[96] That model contains an exponential term that uses the time between patent grant and forward citation to measure the depreciation of knowledge over time.[97] Our exponential weighting methodology comports with this existing literature on the importance of recent citations. We propose that exponential weighting of forward citations is also appropriate in the case of SEPs for several reasons.

First, it is reasonable to expect that patents essential to the same standard cover similar technologies. Further adjusting the weighting of patents relative to their assigned NBER technology category (or some other preassigned technological category) might be unnecessary in this case and might introduce artificial differences between otherwise similar technologies. The importance of this exclusion should be examined by considering the technical coverage of the standard.

Second, standards can be developed quickly, and our proposed citation-weighting measure rewards companies that quickly develop and patent follow-on technology related to the standard. An exponential weighting captures the importance of the knowledge transferred from one patent to future inventors and values the speed of that transfer. Later patents might themselves be declared essential to the standard or to a later standard that the same SSO is developing. Our measure rewards patents that are *both* highly cited and cited more quickly—that is, precisely the patents that we would expect would spur the greatest incremental innovation.

One can also weight forward citations using information about the citing patent. Two methods are commonly used to perform this calculation: (1) comparing the technology categories of the cited patent and the citing patent, and (2) comparing the assignees of the cited patent and the citing patent. We are working to incorporate this information into our future research; however, we discuss the theoretical issues here.

[95] Erjia Yan & Ying Ding, *Weighted Citation: An Indicator of an Article's Prestige*, 61 J. Ass'n Info. Sci. & Tech. 1635 (2010).
[96] Hall, Jaffe & Trajtenberg, *The NBER Patent Citations Data File: Lessons, Insights and Methodological Tools*, *supra* note 60.
[97] *Id.* at 33.

A self-citation is a citation in a new patent to an older patent owned by the same company or inventor. Whether it is necessary to adjust patent-citation scores further to punish or reward self-citations will vary according to the opinion of the researcher and the facts of the case. In some situations, a patent holder will add a self-citation to boost a patent's relative value. In other cases, self-citations can indicate that the patent holder found its initial invention valuable enough to justify its pursuit of follow-on innovation. Our calculations in this article do not account for self-citations, although we intend to investigate the effects of this adjustment in subsequent research.

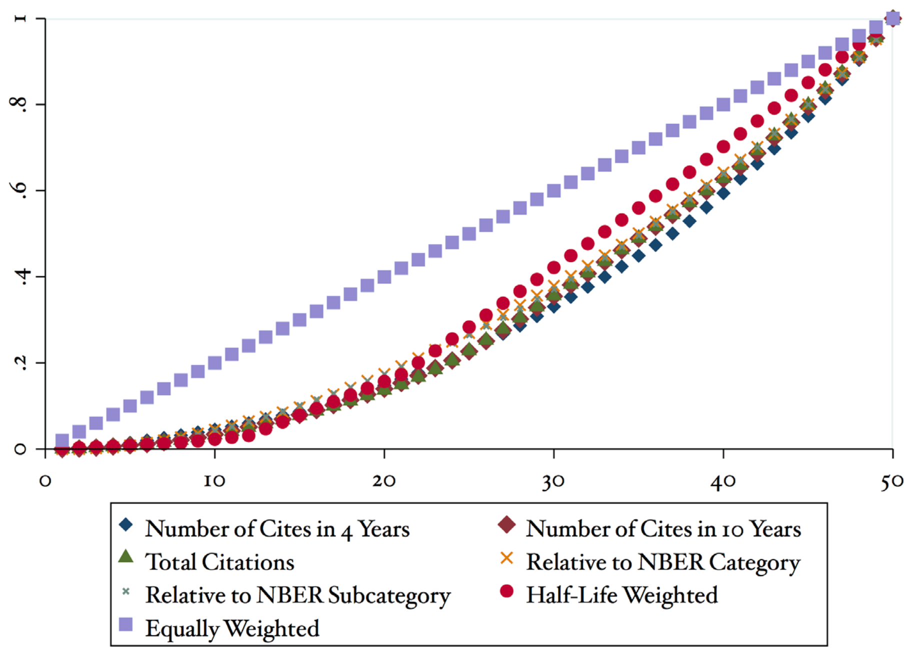
IV. Comparative Analysis of Patent -Valuation Methodologies

We examine the differences in patent valuation between the weighting methods described in Part II using several statistical techniques. The main method that we use is to draw a Lorenz curve, which graphically displays the distribution of value among patents within a standard. From the Lorenz curve, we derive the Gini coefficient for each distribution. We also examine the 90/10 ratio, which measures the value of the patent at the 90th percentile in the distribution versus the value of the patent at the 10th percentile in the distribution.[98]

Using each patent-ranking method, we calculate a score for each patent. We then use these scores to draw the Lorenz curve, shown in Figure 13.

[98] For examples of the use of the 90/10 ratio, see *Measuring Inequality*, World Bank, http://go.worldbank.org/3SLYUTVY00; 2 Helena Afonso, Marcelo LaFleur & Diana Alarcón, United Nations Department of Economic and Social Affairs, Inequality Measurement: Development Issues (2015), http://www.un.org/en/development/desa/policy/wess/wess_dev_issues/dsp_policy_02.pdf.

Figure 13. The Lorenz Curves for Each Patent-Valuation Calculation



Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.
Note: We exclude three expired patents (U.S. Patent Nos. 6,820,181, 7,349,277, and 7,366,920) for purposes of Figure 13. When apportioning a standard's value to patents, one should account for expired patents and patent applications for reasons explained in Sidak & Skog, *Hedonic Prices and Patent Royalties*, *supra* note 4, at 639–40.

The Lorenz curve is constructed by ranking each patent according to a valuation method. The patents are then ordered from least valuable to most valuable. The horizontal axis denotes the SEPs with the least valuable located on the left side of the graph and the most valuable located on the right side of the graph. The valuations are then summed so that the vertical height of the graph is the cumulative portion of the share of value for all patents that are less than or equal to the value of a specific patent. For example, the 10th most valuable patent will be assigned a vertical value of the 10 patents, up to and including that patent. The slope of the Lorenz curve at any point then equals the incremental value added by another patent.[99] For large portfolios, the horizontal axis is sometimes drawn in percentile terms (as is the vertical axis), but for the 50 active U.S. patents declared essential to the LRDIMM standard, we simply use the absolute numbers of the patents themselves, rather than converting to percentiles.

[99] *See* Jonathan Putnam, The Value of International Patent Rights (Feb. 3, 1997) (unpublished Ph.D. dissertation, Yale University), http://www.competitiondynamics.com/wp-content/uploads/International-Patent-Rights.pdf.

The straight line in Figure 13 denotes the line of perfect equality. It shows the Lorenz curve when all patents have equal values and each is assigned an equal share of the standard's value. The deviation of each other Lorenz curve from this line is a measure of the inequality among patent valuations. The flatter the line, the more the SEPs are assigned equal values; conversely, the more curved the line, the more unequal are the valuations assigned to the SEPs. The Gini coefficient measures the deviation of each Lorenz curve from the line of perfect equality. Table 5 reports the Gini coefficients and the 90/10 ratios for each of the seven patent-valuation methodologies we examine.

Table 5. Measures of Inequality for Seven
Alternative Patent-Valuation Methodologies

Valuation Method	Gini Coefficient	90/10 Ratio
Patent Count	0	1
Half-Life Method	0.280	16.182
NBER Technology Category	0.314	7.341
NBER Technology Subcategory	0.318	7.602
Citation Count Weighting	0.345	12.765
Number of Citation, 10-Year Cutoff	0.346	12.706
Number of Citation, 4-Year Cutoff	0.363	8.250

Source: *Data Query*, PatentsView, http://www.patentsview.org/query/; authors' analysis.

The patent-counting method shows the value that each score takes under the assumption that all patents are equally valuable. When all patents are assigned equal values, the Gini coefficient is zero. The Gini coefficient increases to one as the inequality among patents increases—that is, as a few patents contain most of the standard's value. We see that the half-life method produces a relatively more-equal score (according to the Gini coefficient) than do other ranking methods, whereas applying a strict 4-year citation cutoff produces the most unequal valuation. Because we observe very few forward citations that occur after 10 years for the patents in the LRDIMM standard, counting all forward citations (with no further adjustment) produces a result similar to the 10-year cutoff.

The 90/10 ratio shows the inequality between the most-valuable patents and the least-valuable patents. The patent count score has a value of 1 because the patents at the 90th and 10th percentiles are equally valuable. A higher 90/10 ratio indicates a greater relative difference in value between the most valuable patents and the least valuable patents. For example, a 90/10 ratio of 3 would indicate that the patents in the 90th percentile are 3 times as valuable as patents in the 10th percentile, whereas a 90/10 ratio of 30 would indicate that the patents in the 90th percentile are 30 times as valuable as patents in the 10th percentile. Here the results show that the half-life citation weighting produces the highest 90/10 ratio. That is, this particular citation-weighting method produces the greatest difference in relative value between patents in the 90th percentile and patents in the 10th percentile. In contrast, the NBER Technology Category and Subcategory weighting methods yield the least difference between the most and least valuable patents.

The differences between the two measures of patent-value inequality arise because of the shape of the Lorenz curves. For example, the half-life citation weighting has a relatively equal Gini coefficient and a relatively unequal 90/10 ratio because the Gini coefficient is affected more by patents in the middle of the distribution, whereas the 90/10 ratio is more affected by patents at the ends of the distribution. That is, the half-life weighting method assigns low values to low-ranked patents and high values to highly ranked patents, whereas the patents in the middle of the distribution are relatively equal. In contrast, the 4-year citation cutoff assigns relatively low scores to patents in the middle of the distribution, but the highest-valued patents are not much more highly valued than the lowest-valued patents. That is, most patents have relatively low scores, but the most-valued patents are only 8 times as valuable as the least-valuable patents, rather than 12 or 16 times as valuable, as in the cases of some of the other patent-valuation methodologies summarized in Table 5.

Conclusion

In patent-infringement litigation involving standard-essential patents, one must apportion the value of the patents in suit by deriving an appropriate measure of each patent's value relative to the value of other patents that are also declared essential to the standard. Using data on patents declared essential to the LRDIMM standard, we have analyzed multiple methodologies that purport to measure the relative value of patents. We conclude that the choice of a particular patent-valuation methodology is secondary to the apportionment inquiry. In other words, we find that the particular weighting method that a researcher chooses to use is of secondary importance to the researcher's decision to use *some* weighting method, rather than none. A simplistic patent-counting methodology that assigns each patent equal value relies on assumptions that are rarely satisfied in the real world. It produces a result that meaningfully differs from the results of any of the methods that rely on forward citations to measure a patent's value.

We propose a half-life citation-weighting method that researchers might decide to use, in addition to adjustments for technology fields or unweighted citation counts. By placing greater weight on more recent citations, our proposed method attempts to account for the increasing number of citations and patents over time and the importance of speed during the standard setting process. Our proposed method might be particularly appropriate for standards in which the declared-essential patents cover similar technologies or in cases where innovation (and, consequently, the standards-development process) occurs rapidly.

Appendix I. Examples of Patents in NBER Technology Category 2 That Belong in Different NBER Technology Subcategories

U.S. Patent No.	Assignee	Filing Date	Grant Date	NBER Technology Category	NBER Technology Subcategory
8,001,434	Netlist Inc.	Apr. 13, 2009	Aug. 16, 2011	2	22
Title: Memory board with self-testing capability					
Abstract: A self-testing memory module includes a printed circuit board configured to be operatively coupled to a memory controller of a computer system and includes a plurality of memory devices on the printed circuit board, each memory device of the plurality of memory devices comprising data, address, and control ports. The memory module also includes a control module configured to generate address and control signals for testing the memory devices. The memory module includes a data module comprising a plurality of data handlers. Each data handler is operable independently from each of the other data handlers of the plurality of data handlers. Each data handler is operatively coupled to a corresponding plurality of the data ports of one or more of the memory devices and is configured to generate data for writing to the corresponding plurality of data ports.					
7,644,363	Autodesk, Inc.	Apr. 10, 2006	Jan. 5, 2010	2	22
Title: "For-each" label components in CAD drawings					
Abstract: Embodiments of the invention provide a method for generating labels for drawing elements of a computer aided design (CAD) drawing that may include some unknown number of associated or related drawing elements. The method includes receiving a selection of a set of drawing elements in the CAD drawing that are associated with the first drawing element, receiving a selection of fields specifying attributes of the drawing elements in the set of drawing elements to use in generating label content for a label of the first drawing element, and defining a label style based on the selection of the set of drawing elements and the selection of attributes of the of the drawing elements in the set of drawing elements.					

U.S. Patent No.	Assignee	Filing Date	Grant Date	NBER Technology Category	NBER Technology Subcategory
8,225,415	Mitsubishi Electric Corp.	Aug. 24, 2006	July 17, 2012	2	25

Title: Content distribution system, terminal, and server

Abstract: A terminal writes first encrypted data in which a communication key and an owner ID are encrypted by using a public key into a second recording medium, and this medium is mounted to a communication apparatus so that the first encrypted data are transmitted to a server. The server sends second encrypted data which it acquires by encrypting a content decryption key by using the communication key which the server acquires by decrypting the received first encrypted data by using a secret key to the communication apparatus, and causes the communication apparatus to record them into the second recording medium. The terminal decrypts a content stored in a first recording medium by using the content decryption key which it acquires by decrypting the second encrypted data read from the second recording medium by using the communication key.

U.S. Patent No.	Assignee	Filing Date	Grant Date	NBER Technology Category	NBER Technology Subcategory
8,072,837	Netlist Inc.	Dec. 29, 2010	Dec. 6, 2011	2	24

Title: Circuit providing load isolation and memory domain translation for memory module

Abstract: A circuit is configured to be mounted on a memory module configured to be operationally coupled to a computer system. The memory module has a first number of ranks of double-data-rate (DDR) memory devices configured to be activated concurrently with one another in response to a first number of chip-select signals. The circuit is configurable to receive a set of signals comprising address signals and a second number of chip-select signals, the address signals comprising bank address signals. The circuit is further configurable to monitor command signals received by the memory module, to selectively isolate a load of at least one rank of the first number of ranks from the computer system in response to the command signals, and to provide the first number of chip-select signals to the first number of ranks in response at least in part to the received bank address signals and the received second number of chip-select signals.

U.S. Patent No.	Assignee	Filing Date	Grant Date	NBER Technology Category	NBER Technology Subcategory
7,643,247	TDK Corp.	Dec. 5, 2006	Jan. 5, 2010	2	24

Title: Beta-phase tantalum thin-film resistor and thin-film magnetic head with the resistor

Abstract: A thin-film resistor that has a stable electric resistance, the phase transformation to the alpha-phase being suppressed even in the high temperature environment, is provided. The thin-film resistor has a layered structure of: a base layer formed of a double-layered film in which an alloy film containing nickel and copper, an alloy film containing nickel and chromium or an alloy film containing copper and manganese is stacked on a tantalum film, or formed of a single alloy film containing nickel and chromium; and an electric resistance layer formed of a beta-phase tantalum film or an alloy film mainly containing beta-phase tantalum, and deposited on the base layer, the electric resistance layer having a crystal structure in which (002) plane of the beta-phase crystal is most strongly oriented to the layer surface.

U.S. Patent No.	Assignee	Filing Date	Grant Date	NBER Technology Category	NBER Technology Subcategory
7,652,661	Avago Technologies ECBU IP PTE.	Dec. 23, 2004	Jan. 26, 2010	2	23

Title: "Seeing eye" mouse for computer system

Abstract: A hand operated pointing device for use with a computer includes a movable housing, a source of non-coherent light illuminating a work surface and circuitry in the movable housing using arrays of data related to light reflected by the illuminated work surface to produce values by processing portions of a first array with portions of a second array. One of the values may be identified to represent movement of the housing relative to the work surface. The light may illuminate surface irregularities at an angle of incidence low enough to produce suitable arrays of data for processing from highlights and shadows of the illuminated surface irregularities. The circuitry may produce additional values by processing another selected portion of the first array with portions of a third array or may select a fourth array for processing with the third array. Predictions may be derived from the values.