THE
CRITERION
JOURNAL ON INNOVATION

VOL. 8 2026

John Nash's Bargaining Parabola

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Imagine that the following question is on the final exam in a microeconomics course. Two persons bargain over the sale of a widget. Suppose that the transaction, if it occurs, creates a surplus of ten. Those ten units of value exist if the parties complete a trade, but vanish if the parties fail to come to terms. How much of the ten goes to the buyer, and how much to the seller?

I. The Nash Parabola

John Nash of Princeton University, who shared the Nobel Prize in economics in 1994, addressed that question in *The Bargaining Problem* in 1950, at age 21, with a proposition of striking economy: among the feasible divisions, select the one that maximizes the product of the two parties' gains above their disagreement payoffs.[1] In 1953, Nash extended his 1950 article in *Two-Person Cooperative Games* in a manner that "tells the players what threats they should use in negotiating."[2] A party's disagreement payoff is what he would receive if the negotiation failed and no agreement were struck—that is, a party would receive his next-best outcome away from the table. Nash called that point the "anticipation," and he took it as a datum of the bargaining problem rather than a result of it.[3] It is also called the "threat point."[4]

To divide the surplus of ten, the Nash bargaining solution maximizes the multiplicative product of the parties' gains. Multiply the buyer's share of the gains above his disagreement payoff by the seller's respective share of the same, and—in the symmetric case—each party receives a surplus of five. I have examined elsewhere the consequences of that proposition for the adjudication of reasonable-royalty disputes concerning patents, and the refusal of

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[1] John F. Nash, Jr., *The Bargaining Problem*, 18 Econometrica 155, 157–59 (1950).

[2] John Nash, *Two-Person Cooperative Games*, 21 Econometrica 128, 130 (1953).

[3] Nash, *The Bargaining Problem*, *supra* note 1, at 156–57.

[4] Nash, *Two-Person Cooperative Games*, *supra* note 2, at 133–34.

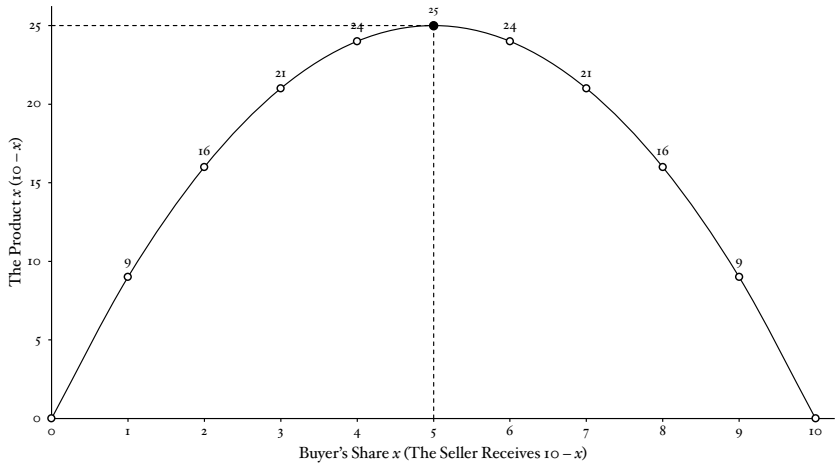
the U.S. Court of Appeals for the Federal Circuit to permit an expert witness to substitute the Nash bargaining solution for evidence about the actual patent-infringement case at hand.[5]

The two disagreement payoffs in Nash's formulation are boundaries, and the same two boundaries appear in the law of reasonable royalties as the licensor's minimum willingness to accept and the licensee's maximum willingness to pay, which mark the ends of the bargaining range. Each point identifies the royalty at which one party would be indifferent between reaching an agreement and walking away from the negotiation.[6] The two framings correspond where utility is transferable and the parties are risk-neutral.

Nash supplied one method for determining a point within the range. The alternative that I have pursued elsewhere measures where within that range the parties would have come to rest during a hypothetical negotiation. This article returns to Nash's proposition itself, and to a feature of it that I have not seen scrutinized: that the multiplication at its heart does more than identify a bargaining solution. It embeds an equal-weighted judgment about the relative importance of one person's gain and another's.

Lay out the product of the two parties' gains across the range of possible divisions. If the buyer takes one and the seller nine, the product is nine; two and eight yields 16; three and seven, 21; four and six, 24; five and five, 25; and thereafter the multiplicative product declines through the same parabolic steps by which it rose. The objective function that the Nash bargaining solution maximizes is a parabola, depicted in Figure 1, and the even split sits at the parabola's vertex of 25.

Figure 1: Nash's Parabola of Multiplicative Surplus



[5] *See* J. Gregory Sidak, *Bargaining Power and Patent Damages*, 19 Stan. Tech. L. Rev. 1 (2015).
[6] *See id.* at 10–17 (defining the bargaining range and its boundaries).

II. Why Multiply and Not Add?

But why a multiplicative product at all? Why should the measure of a good bargain be the buyer's share multiplied by the seller's share, rather than the two shares added together? Addition is the natural arithmetic of aggregation; one would expect the combined value to be the sum of what each party receives, which for a fixed pie of ten is always ten, whatever the division. That is precisely the difficulty. Addition cannot discriminate among divisions of a fixed surplus, because every division of ten sums to ten; the sum is blind to distribution. The multiplicative product is not. Multiplication rewards equality—five times five exceeds one times nine—and so a criterion built upon multiplication will always select an interior split rather than a corner. It is the multiplicative form, and not the arithmetic of aggregation, that gives the criterion any distributive content at all.

Nash, however, did not simply assert the multiplicative product. He asserted something that appears more modest and proves to be more consequential: a short list of axioms, from which the multiplicative product follows as the unique survivor. Two of those axioms deserve the scrutiny that they seldom receive in a courtroom.

A. Invariance and Symmetry

The first axiom is invariance to the scale in which each party's utility is measured. To see what that axiom demands, one must first understand what a utility number is and what it is not. A von Neumann-Morgenstern utility does not measure satisfaction. It is a bookkeeping device recovered from a person's own choices among gambles. In the familiar normalized construction, one assigns the number zero to the person's worst outcome and the number one to his best; each intermediate outcome then receives the probability of the best outcome that would leave him indifferent between the certainty of that intermediate outcome and a gamble between the two extremes.[7] But the zero and the one are conventions of normalization, not facts discovered in the world. Had one begun instead with 100 and 350, one would have recovered an entirely different set of numbers describing the identical person with the identical preferences. The preferences would not have changed; only the utility scale would have.

The family of relabelings that leaves those preferences intact is exactly the family of positive affine transformations: replace the utility function

[7] John von Neumann & Oskar Morgenstern, Theory of Games and Economic Behavior 15–31 (Princeton Univ. Press 1944). The axiomatic treatment of utility appears in the appendix to the second edition, John von Neumann & Oskar Morgenstern, Theory of Games and Economic Behavior 617–32 (Princeton Univ. Press 2d ed. 1947).

u with $au + b$, for any constant b and any constant a exceeding zero.[8] The constant b moves the origin, the constant a changes the size of the unit, and the requirement that a be positive keeps better outcomes ranked above worse ones. The family is no wider than that. An order-preserving (but not affine) relabeling—squaring the numbers, for instance—would preserve the ranking of certain outcomes but would destroy the ranking of gambles, and the construction exists to capture the ranking of gambles. A positive affine transformation therefore preserves that ranking, while leaving the unit of measurement free. The family is wide enough that the zero and the unit are conventions, and narrow enough that expected values retain their economic meaning. Nash's construction fixes the constant b at the outset, by measuring each party's utility as a gain above his disagreement payoff, which places both origins at the point of no agreement. Only the unit remains variable.

A change of unit leaves a limited but precise invariant. Within a single person, the ratio of two utility differences is meaningful: if the improvement from A to B is twice the improvement from C to D on one admissible scale, it is twice on every admissible scale. The size of the unit itself, however, has no invariant meaning—and from that characteristic follows the premise that governs everything after it. Each party's scale is fixed by a convention adopted for him alone, so that one person's utility and another person's utility are not commensurable. The buyer's utility gain of five and the seller's utility gain of five are readings taken from two separately calibrated instruments. The familiar case is temperature.

Celsius and Fahrenheit are positive affine transformations of one another, and each describes the same physical facts faithfully. One may say that the rise from morning to noon was twice the rise from noon to evening, and the statement holds on either scale. One may not say that today is twice as warm as yesterday, because the zero was put where it is by Anders Celsius and by Daniel Fahrenheit rather than by nature. Least of all may one multiply a Celsius reading in one room by a Fahrenheit reading in another and expect the multiplicative product to describe anything at all.

The analogy is in fact too generous to the bargaining theorist, for behind the two thermometers there stands a single physical quantity, and one may convert between the scales. Behind two utility functions there stands no common quantity and no conversion. The impossibility is not a shortage of evidence that further discovery might repair; it is a consequence of whence came the numbers used. A person's choices among his own gambles determine his own scale and disclose nothing whatsoever about anyone else's. Whatever criterion a court is petitioned to apply, therefore, must return the same physical division of the widget when every one of the buyer's utility

[8] *See, e.g.*, Hal R. Varian, Microeconomic Analysis 482 (W. W. Norton & Co. 3d ed. 1992) (explaining affine functions).

numbers is multiplied by ten, for nothing about the buyer, the seller, or the widget has changed.

The invariance axiom therefore requires the bargaining solution to remain unchanged when either party rescales his utility. The multiplicative product satisfies that requirement: if the buyer multiplies his utility by any positive constant, the entire product is multiplied by that constant, and the division that maximizes it does not move. The sum does not satisfy it: rescale one party's utility and the maximizing division changes in favor of the party whose utility scale was enlarged. Nash's axioms, taken together, and not scale invariance alone, select the multiplicative product. The multiplicative product follows because the axioms forbid treating one party's utility unit as commensurable with the other's, and the multiplicative product survives a separate rescaling of either party's utility.

The second axiom, symmetry, then fixes the weights at equality and delivers the even split as a theorem. The egalitarian result that appears to issue from pure mathematics was placed into the premises by the choice of a multiplicative criterion and an axiom of symmetry; the bargainers themselves—their histories, their alternatives, their urgencies—had nothing to do with the result.

B. The Missing Daubert *Objection*

Nothing in the preceding analysis depends on rejecting Nash's theorem, which is true within its mathematical premises. The problem begins when a damages expert treats those premises as facts about an actual bargain.

My claim is narrower. A Nash-based damages opinion should survive *Daubert*[9] only if the expert discloses and defends the equal weighting that the multiplicative division of surplus assumes.

Nash's theorem never unsettled me. The faith of the theorem's admirers is another matter. The economists who reached for the Nash bargaining solution in litigation did so precisely because the solution promised a damages methodology that would survive a *Daubert* challenge in a jury trial—and, in a bench trial, would survive a motion *in limine* because the expert could present the damages figure as the output of a theorem rather than as an asserted rule of thumb. But the multiplicative division of surplus in the Nash bargaining solution is not now, and never has been, a finding of fact. The relevant *Daubert* question should therefore be whether the expert has derived the proposed division of surplus from case-specific evidence, or instead has merely assumed that division of surplus through the normalization and symmetry choices embedded in the Nash bargaining solution. A method embraced for its appearance of rigor rests, at its foundation, upon its least

[9] Daubert v. Merrell Dow Pharms., Inc., 509 U.S. 579, 589–95 (1993) (Blackmun, J.).

rigorous move: an equal weighting of the two parties' welfare, assumed rather than derived.

My objection to the use of the Nash bargaining solution is largely unmade because a simpler objection became available. After *Uniloc*[10] had swept away the 25-percent rule of thumb, the Nash bargaining solution entered the courtroom as its more rigorous-seeming successor. *VirnetX* then gave opponents a ready answer. The Nash bargaining solution might be sound, but the theorem itself says nothing about whether the facts of a particular dispute fit its premises.[11] With that objection available, no litigant needed to press the deeper point that the equal division itself had been baked into the assumptions of the model.

III. The Courts' Discipline

The even split of surplus is not the only assumption that the Nash bargaining solution carries, and in *VirnetX* the Federal Circuit held those assumptions to account. This Part traces that discipline to its root, beginning with an assumption on which the Nash bargaining solution depends: that the pie is fixed in size before the parties divide it.

A. The Fixed Pie

The standard Nash bargaining problem takes two things as given: the disagreement point, and the feasible set, which is the collection of every division of the surplus that the parties could reach by agreement. Those two givens are precisely what many consequential disputes contest. The parabola is drawn on the assumption that the ten units of value already exist and lie fixed upon the table. The parabola can model the division of that stipulated surplus. But the parabola cannot explain how the surplus came into being, nor can the parabola show how the feasible set would change had the innovation never been made. To change the size of the pie is not to move along the parabola but to substitute a different parabola, and the even split that the Nash bargaining solution prescribes says nothing whatsoever about that substitution. Creative destruction does not redistribute the area beneath a particular parabola. Instead it defines an entirely new parabola.

There is a further sense in which the parabola misleads, for the surplus that the parabola lays upon the table is not the surplus that the inventor created.

Kenneth Arrow, who would receive the Nobel Prize a decade later, explained in 1962 why a competitive market will not allocate resources to

[10] Uniloc USA, Inc. v. Microsoft Corp., 632 F.3d 1292, 1315 (Fed. Cir. 2011) (Linn, J.).
[11] VirnetX, Inc. v. Cisco Sys., Inc., 767 F.3d 1308, 1332 (Fed. Cir. 2014) (Prost, C.J.).

invention efficiently.[12] Invention, Arrow reasoned, is the production of information, and information has three properties that defeat the ordinary market. Information is indivisible: the cost of producing information does not vary with the number of persons who use it, so a price set at marginal cost returns nothing to the producer.[13] Information is inappropriable: once the inventor discloses what he knows, he cannot prevent others from using it, and disclosure is the condition of any sale.[14] And invention is uncertain: the inventor commits his resources before he knows whether anything of value will emerge, and no market insures him against that risk.[15] The second property governs the argument here. The value that an invention creates is largely social, and only a portion of that value returns to the inventor as private and appropriable gain.[16]

William Nordhaus, who would receive the Nobel Prize in 2018, measured the size of that portion.[17] Nordhaus asked what fraction of the gains from technological progress the innovator keeps, and he answered the question for the American economy as a whole. He defined the innovator's share as Schumpeterian profits, which are the profits that arise when a firm is able to appropriate the returns from innovative activity, and he adopted a deliberately restrictive version of that definition, counting only the profits that exceed the risk-adjusted return on the innovative investment. Research and development that earns a normal return therefore yields no Schumpeterian profit at all.[18] He defined the denominator geometrically. For an innovation that lowers the cost of production, the social surplus is the area bounded by the price that the displaced technology commanded, the demand curve, and the cost of the new technology. The innovator's profit is a rectangle lying inside that area, and the remainder passes to buyers in the form of lower prices.[19] Nordhaus then estimated the ratio of the one to the other for the United States nonfarm business sector over the years 1948 to 2001, working from the relation between the growth of multifactor productivity and the margin of property income over cost. He excluded farming because land is so large a share of the capital employed in that sector.[20] His central estimates were a first-year appropriability of about 7 percent and a decay of Schumpeterian profits of about 20 percent per year. Combined with economic growth of

[12] Kenneth J. Arrow, *Economic Welfare and the Allocation of Resources for Invention*, *in* The Rate and Direction of Inventive Activity: Economic and Social Factors 609 (Princeton Univ. Press 1962).
[13] *Id.* at 614–15.
[14] *Id.* at 615.
[15] *Id.* at 613, 616.
[16] *Id.* at 619.
[17] William D. Nordhaus, *Schumpeterian Profits in the American Economy: Theory and Measurement* (Nat'l Bureau of Econ. Rsch., Working Paper No. 10433, 2004).
[18] *Id.* at 4.
[19] *Id.* at 7–8 & fig. 1.
[20] *Id.* at 18–20.

3 percent per year and a discount rate of 10 percent per year, those estimates imply that innovators capture about 2.2 percent of the present value of the social returns to innovation.[21] Across his specifications the ratio ranges from 1.3 percent to 3.3 percent.[22] Nordhaus concluded that "only a miniscule fraction of the social returns from technological advances over the 1948–2001 period was captured by producers," and that most of the benefits of technological change are passed on to consumers instead.[23]

That economy-wide estimate does not allocate surplus within any particular licensing negotiation or supply chain. The "pie" that two litigants divide is only a residual fragment of a much larger social gain—a gain that the innovator brought into being. Splitting that fragment evenly treats the fragment as the whole, and treats as merely found a thing that one party created.

B. The Record in the Courts

The danger that Arrow and Nordhaus identified was not hypothetical. It has already decided cases.

1. VirnetX *and Its Progeny*

In *VirnetX* itself, the patent holder's damages expert, Roy Weinstein, valuing Apple's allegedly infringing FaceTime feature, treated the incremental profit associated with that feature as a fixed pie and began from the even split that the Nash bargaining solution prescribes; Weinstein then adjusted that even split to 55 percent and 45 percent in Apple's favor to reflect what he took to be Apple's superior bargaining position, arriving at a figure of $588 million, with a survey-based parallel approach reaching a comparable $606 million.[24] The pool itself was estimated from revenue attributed to the addition of a front-facing camera.

The Federal Circuit rejected Weinstein's method. The Nash bargaining solution, the court observed, "asserts nothing about what situations in the real world fit those premises."[25] The Federal Circuit added that beginning from an arbitrary even split and then adjusting it by case-specific factors yields a conclusion no less arbitrary than the abandoned 25-percent rule of thumb to which the court likened the solution.[26] The expert had assumed the

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[21] *Id.* at 22.

[22] *Id.*; *see also id.* at 23 tbl. 1, 25 tbl. 2.

[23] *Id.* at 34; *see also* William D. Nordhaus, Invention, Growth, and Welfare: A Theoretical Treatment of Technological Change (M.I.T. Press 1969).

[24] *VirnetX*, 767 F.3d at 1325–26.

[25] *Id.* at 1332.

[26] *Id.* at 1331–33. For further analysis of the *VirnetX* decision, see Sidak, *Bargaining Power and Patent Damages*, *supra* note 5, at 6–10; J. Gregory Sidak, *The Proper Royalty Base for Patent Damages*, 10 J. Competition L. & Econ. 989, 1014–17 (2014).

existence and size of the pie and the manner of its division, when the size and the very existence of that pie—the surplus attributable to the patented feature—were the questions that the litigation had been convened to decide.

The discipline of *VirnetX* has held since 2014. The courts have continued to turn away reasonable-royalty opinions that rest on the even split that the Nash bargaining solution had been portrayed as prescribing. The most visible repudiation is *Bayer HealthCare LLC v. Baxalta Inc.*, where the district court excluded Sumanth Addanki's opinion that the reasonable royalty equaled the midpoint of a bargaining range derived from the Nash bargaining solution, and the Federal Circuit later recounted that exclusion while affirming the jury's award of a 17.78-percent royalty—a figure that the jury fixed upon the evidence, without the even split that the Nash bargaining solution would have supplied.[27]

2. The Provenance of the Method

The application of the Nash bargaining solution to reasonable royalties did not descend from the refereed economics scholarship on bargaining. One important route into patent litigation was the practitioner article by economists William Choi and Roy Weinstein entitled *An Analytical Solution to Reasonable Royalty Rate Calculations*, published in 2001 in a student-edited intellectual-property journal.[28]

The Choi-Weinstein article is abler than its courtroom caricature. The article transcribes Nash's mathematics faithfully, states his axioms in plain terms, and carries the solution through to a closed-form per-unit royalty that a damages expert could actually compute. That formula does not conceal that it produces an even split. Equation 17 in the Choi-Weinstein article sets the royalty at one-half of the gap between price and the licensee's average total cost, less half of the licensee's opportunity cost per unit, and the authors state the point in words: the royalty rate "should be established at one-half of the difference between the price and the licensee's average total cost."[29] Equation 23 in the Choi-Weinstein article later adds a term for the parties' relative disagreement payoffs, leaving the one-half division in place.[30]

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[27] Bayer HealthCare LLC v. Baxalta Inc., 989 F.3d 964, 984 (Fed. Cir. 2021) (Stoll, J.), *aff'g in relevant part* No. 16-cv-1122, 2019 U.S. Dist. LEXIS 11952, at *21–22 (D. Del. Jan. 25, 2019) (Andrews, J.); *see also* Good Tech. Corp. v. MobileIron, Inc., No. 5:12-cv-05826-PSG, 2015 U.S. Dist. LEXIS 87347, at *21–22 (N.D. Cal. July 5, 2015) (Grewal, Mag. J.). The district courts had begun this discipline before *VirnetX*. *See, e.g.*, Suffolk Techs., LLC v. AOL Inc., No. 1:12-cv-00625, 2013 U.S. Dist. LEXIS 64630, at *5–6 (E.D. Va. Apr. 12, 2013) (Ellis, J.); Dynetix Design Sols., Inc. v. Synopsys, Inc., No. 5:11-cv-05973, 2013 U.S. Dist. LEXIS 120403, at *15 & n.34 (N.D. Cal. Aug. 22, 2013) (Grewal, Mag. J.); Oracle Am., Inc. v. Google Inc., 798 F. Supp. 2d 1111, 1119–20 (N.D. Cal. 2011) (Alsup, J.).

[28] William Choi & Roy Weinstein, *An Analytical Solution to Reasonable Royalty Rate Calculations*, 41 IDEA 49 (2001).

[29] *Id.* at 58.

[30] *Id.* at 59.

Choi and Weinstein therefore adopt the symmetric Nash bargaining solution rather than an independent measure of the parties' division of surplus. Relative bargaining power enters only through the parties' disagreement payoffs; the surplus lying above those payoffs is divided equally in every case that Choi and Weinstein consider. They argue that the parties first receive their disagreement payoffs and then "split the remaining profits equally," and that the bargaining surplus is divided "down the middle."[31] Choi and Weinstein further assert that "[i]f both sides have equal disagreement payoffs, then additional profits achieved from licensing are split equally."[32]

But the disagreement payoffs are not given by the Nash bargaining solution. Nash takes them as data of the problem; Choi and Weinstein concede that the "exact functional form of these disagreement payoffs depends on specific assumptions about the two firms and the economic conditions present at the time of disagreement[,]"[33] and then supply those assumptions themselves—setting the licensor's payoff at zero in one case and at the profit of a high-cost sole producer in another.[34] Choi and Weinstein define the disagreement payoff of each party as the profit that it "expects to receive if the negotiation fails."[35]

Yet the article does not scrutinize its own recommendation. The Choi-Weinstein article presents the solution as one that yields, in its authors' words, an "efficient and fair" result—importing the word "fair" as though the multiplicative product of surplus payoffs carried no distributive commitment—and rests the derivation on the simplifying premise that the licensee is a monopolist.[36] The article's ambition was practical: to spare the damages expert the *ad hoc* weighting of the *Georgia-Pacific* factors. The Choi-Weinstein article lists the symmetry condition among the axioms of the Nash bargaining solution—if the parties' disagreement profits are equal, the bargaining solution must treat them equally—and later Choi and Weinstein apply the equal-split implication, but their article does not examine the welfare judgment embedded in that assumption of symmetry. Nor do Choi and Weinstein dwell on the assumptions that allow monetary profits to serve as the common scale for the parties' gains. In a footnote, Choi and Weinstein observe that the assumption of transferable payments "can guarantee the given scale factors in a game will also be the natural scale factors for the NBS [Nash bargaining solution]," and that risk neutrality is

[31] *Id.* at 55.
[32] *Id.* at 59.
[33] *Id.* at 54.
[34] *Id.* at 57–58.
[35] *Id.* at 54.
[36] *Id.* at 49 ("efficient and fair"); *id.* at 57 (stating that "the joint profit from licensing is equal to the monopoly profit gained from commercializing the patented invention").

"also an important assumption."[37] Choi and Weinstein then conclude, in the same sentence, that "in the context of firms negotiating over an agreement, the assumption is plausible."[38] The premise upon which the comparison of one party's gain with another's wholly depends is thus acknowledged only in passing in a single footnote. The intended readership of the Choi-Weinstein article was unlikely to demand such scrutiny, because the Nash bargaining solution was being offered as a practical tool for organizing damages testimony in patent-infringement litigation rather than as a generalized theory of distributive justice.

Weinstein, the co-author of that article and a testifying damages economist, carried the method from the page into the courtroom. The rejection that followed was neither swift nor unanimous. The trial court admitted the opinion, the jury heard it and returned a verdict, and only on appeal did the Federal Circuit hold that the premises of the Nash bargaining solution had not been shown to fit the case, and liken the method to the 25-percent rule that *Uniloc* had discarded three years earlier.[39] The late Judge T.S. Ellis reached the same destination a year before *VirnetX*, in *Suffolk Technologies v. AOL*, finding Weinstein's even split not adequately tied to the facts of the case.[40] In 2012 in *Mformation Technologies v. Research in Motion*, the year before *Suffolk*, Weinstein's Nash-based analysis survived a *Daubert* challenge, but only because he offered the Nash bargaining solution there not as a stand-alone even split but as a "check" on a *Georgia-Pacific* analysis that he had already performed.[41] Taken together, the cases suggest a distinction between using a Nash-based analysis as a corroborative device and asking it to determine the allocation itself. The former could survive gatekeeping because the distributive judgment had already been supplied by an independently grounded analysis, whereas the latter exposed the method's deeper weakness. Weinstein's difficulty, then, was not a failure of execution. The method could be competently applied and still lack any basis for deciding how the bargaining surplus should be divided.

One feature appears to unite the deployments examined here. Each invoked Nash's 1950 axiomatic solution—the symmetric result that yields an even split in the symmetric case—and none appears to have relied upon the model that Nash published in 1953, in which the division of surplus turns on

[37] *Id.* at 54 n.26.

[38] *Id.*; *see also id.* at 53 n.20 ("More formally, the solution is independent of any numeric risk-neutral utility specification.").

[39] VirnetX, Inc. v. Cisco Sys., Inc., 767 F.3d 1308, 1325, 1331–33 (Fed. Cir. 2014) (Prost, C.J.).

[40] Suffolk Techs., LLC v. AOL Inc., No. 1:12-cv-00625, 2013 U.S. Dist. LEXIS 64630, at *5–6 (E.D. Va. Apr. 12, 2013) (Ellis, J.).

[41] Mformation Techs., Inc. v. Rsch. in Motion Ltd., No. C 08-04990 JW, 2012 U.S. Dist. LEXIS 56784, at *15 & n.19 (N.D. Cal. Mar. 29, 2012) (Ware, J.); *see also* Gen-Probe Inc. v. Becton Dickinson & Co., No. 09-CV-2319 BEN NLS, 2012 U.S. Dist. LEXIS 189388, at *10 n.3 (S.D. Cal. Nov. 26, 2012) (Benitez, J.) (admitting the Nash bargaining solution alongside the expert's real-world observations).

the threats that each party can credibly deploy. Nash's later model would have compelled the expert to measure the litigants' actual leverage, which is to say the very fact that the litigation had been convened to resolve. By taking the 1950 Nash bargaining solution as the baseline and grafting a discretionary adjustment onto it—Weinstein's move from the even half to 45 percent—the analyst kept symmetry as his default and treated asymmetry as a correction, whereas the 1953 model would have made asymmetric bargaining leverage the object of proof. The even split was therefore not read from the parties. The even split was assumed, and then adjusted.

3. The Instructive Case of Iain Cockburn

The most telling instance is not Weinstein's but the one that preceded it. In *Oracle v. Google*, the patent holder's damages expert was Iain Cockburn.[42] He is an economic scholar of the first rank—a Harvard Ph.D., a professor at Boston University, a research associate of the National Bureau of Economic Research, a collaborator of Zvi Griliches, and an authority on the economics of innovation.[43] That such a scholar deployed the division of surplus that the Nash bargaining solution prescribes, and that Judge William Alsup of the Northern District of California excluded that analysis,[44] is not evidence that the method attracts only weak proponents. The exclusion is evidence of the reverse. If an economist of Cockburn's standing might carry the multiplicative split into a federal court without pausing over the equal-weighting choice embedded within it, then the objection stayed buried for want of proximity, not for want of talent. Cockburn's eminence is no embarrassment to the argument pressed here. His eminence is that argument's best proof.

A further irony can be stated precisely. Cockburn's own scholarship concerns appropriability and spillovers—the measurement of how little of the social value of an invention its creator manages to capture—which is the very subject that Arrow and Nordhaus address and that this Part has examined. In his expert report, Cockburn nonetheless treated the patent holder's slice of profit as a fixed pie to be halved.

His expertise bears upon the first of the two objections raised here—that the bargained pie is a residue of a far larger social surplus. On that count a fair-minded reader would fault the framing of the litigation, which fixed the pie, and not the economist who divided it.

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[42] Oracle Am., Inc. v. Google Inc., 798 F. Supp. 2d 1111, 1119 (N.D. Cal. 2011) (Alsup, J.).

[43] *See* Iain Cockburn & Zvi Griliches, *Industry Effects and Appropriability Measures in the Stock Market's Valuation of R&D and Patents*, 78 Am. Econ. Rev. 419 (1988); Rebecca Henderson & Iain Cockburn, *Scale, Scope, and Spillovers: The Determinants of Research Productivity in Drug Discovery*, 27 RAND J. Econ. 32 (1996).

[44] *Oracle*, 798 F. Supp. 2d at 1119 (excluding Cockburn's analysis because he "did not provide an adequate justification for applying the Nash [bargaining] solution to the facts of this case").

The second objection—the equal-weighted welfare judgment embedded in the multiplication—lay outside even Cockburn's wide field. Equal weighting is built into the model's symmetry assumption. The cases therefore show more than a failure to tie the theorem to the facts. They show that the even split entered the analysis before the expert measured the parties' relative bargaining power.

4. Why the Record Is Confined to Patent Litigation

I have not located any reported decision outside patent litigation—in antitrust, breach of contract, or any other field of commercial damages—passing upon the admissibility under *Daubert* of expert testimony that derives a damages figure from the Nash bargaining solution. That confinement appears to reflect procedure rather than the reach of the Nash bargaining solution.

The cluster of reported decisions in patent litigation reflects procedure, not the reach of the Nash bargaining solution. Patent-infringement damages are tried to a jury, so the judge sits as a gatekeeper under *Daubert* and can exclude an unreliable model before the jury may hear expert testimony predicated on it. In contrast, an antitrust suit to enjoin a pending merger seeks an equitable remedy and ordinarily is tried to the bench. Federal Rule of Evidence 702 still applies, but the absence of a jury reduces the practical need for categorical pretrial exclusion; the judge may hear the disputed testimony and decide what weight it deserves. The Nash bargaining solution has been pressed, and doubted, along both routes.

In *United States v. AT&T*, the government's challenge to the Time Warner merger rested on a Nash bargaining model, offered by Carl Shapiro, predicting that the merged firm's added leverage would raise the price of Turner's content to rival distributors.[45] Judge Richard Leon did not reject Nash bargaining models categorically; instead, he rejected the model's prediction because the inputs on which it depended—the rate at which subscribers would switch during a blackout, the profit margins, the cost of the blackout itself—found no support in the record.[46] The U.S. Court of Appeals for the D.C. Circuit affirmed.[47]

C. The Deeper Premise

Those decisions share a common ground: in each, the court refused a bargaining model whose premises had not been tied to the record. That ground is

[45] United States v. AT&T, Inc., 310 F. Supp. 3d 161, 219–25 (D.D.C. 2018) (Leon, J.).

[46] *Id.* at 225–41.

[47] United States v. AT&T, Inc., 916 F.3d 1029 (D.C. Cir. 2019) (Rogers, J.). For analysis of the decision, see Janusz A. Ordover, J. Gregory Sidak & Robert D. Willig, *Is Professor Salop Right That Judge Leon Bungled* United States v. AT&T*?*, 3 Criterion J. on Innovation 249 (2018).

sound, and its principle reaches further than the *VirnetX* court had occasion to say. As explained above, the Federal Circuit observed that the Nash theorem arrives at a result that follows from a certain set of premises but itself asserts nothing about whether those premises hold on the facts of the case at hand.[48] The premises usually identified include Nash's four axioms, one of which is symmetry. In explaining the symmetric case, Nash also stipulated in plain words that the bargainers are "equal in bargaining skill."[49] Equal bargaining skill is exactly what the expert's adjustment from the even split to 55 percent and 45 percent was intended to deny. Inequality of that kind is what brings most litigants into court. To bolt unequal bargaining power onto a derivation that presupposes equal bargaining skill is to deny the premise from which the even split follows.

But beneath those axioms lies a premise deeper still, which no expert defends because it is seldom recognized: that the logarithms of the parties' gains are combined with equal weights. That formulation remains invariant to separate positive rescaling, but the equality of the weights is a normative choice that the mathematics does not recover from evidence about the parties. An expert who offers the even split as the output of a mathematical theorem has not measured the parties' relative bargaining power. He has reported the vertex of a parabola whose shape he assumed, whose symmetry he imposed, and whose equal-weighting choice he has not disclosed. The pie that he divided might itself be only a residual slice of the larger surplus created by the innovation claimed in the patent in suit.

The same one-half returns wherever symmetry is admitted under another name, even when the multiplicative product is stripped away. Model the negotiation not by an axiom but by two probabilities: the licensee's bid function, meaning the likelihood that it will accept a given royalty within some period, and the patent holder's ask function, meaning the likelihood that it will accept the same royalty. The likeliest agreement falls at the even division precisely, and only, when the two functions are mirror images of each other.[50]

The even split is then no more than the arithmetic of that mirror. There is no economic reason to expect the bid function and the ask function to be symmetric. Risk aversion, discount rates, and unequal appetites for delay deform them, and, once deformed, they no longer meet at the center.[51]

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[48] *See supra* note 24 and accompanying text.

[49] Nash, *The Bargaining Problem*, *supra* note 1, at 155.

[50] *See* J. Gregory Sidak, *The Fair Division of Surplus from a FRAND License Negotiated in Good Faith*, *in* 5G and Beyond: Intellectual Property and Competition Policy in the Internet of Things 79, 106–08 (Jonathan M. Barnett & Sean M. O'Connor eds., Cambridge Univ. Press 2024).

[51] *Id.* at 106–08.

The cake-cutting rule of the philosophers—you cut, I choose—arrives at the even split by the identical device, and is fair—as John Rawls,[52] Ken Binmore,[53] and Peyton Young[54] remark—only because the symmetry of the procedure removes any ground for complaint and, in the terms that Binmore and Young use, renders the outcome envy-free.[55] In each of those constructions the two halves of the cake are placed in the premises by an assumption of symmetry and then drawn out of the conclusion as though they had been found there.

Remove the symmetry, and the division is no longer a number but a relation. Between unequal parties, the division of surplus depends chiefly on their relative patience. Bargaining models represent patience by discount factors, which resemble the parties' costs of capital without being the same thing. Other things equal, the party that can better afford to wait—or that has the stronger outside option—commands the larger portion, and the split departs from the center with that advantage. Ariel Rubinstein of Tel Aviv University gave that dependence precise form in his noncooperative theory of bargaining.[56] The Nash bargaining model, having assumed the parties symmetric at the outset, can never reach that refinement. That the two theories are not rivals but two views of one object was settled by Binmore, Rubinstein, and Asher Wolinsky, who proved that the perfect-equilibrium division of the strategic, offer-and-counteroffer game converges upon the Nash bargaining solution as the interval between offers shrinks to nothing—with the shares fixed by the parties' relative impatience.[57] The cooperative axiom and the strategic model therefore agree upon what the even split requires: two equally patient parties. Any asymmetry in patience carries the division away from the center. A judge who must find the division as a fact has to measure the patience and the outside options of the litigants before him. He cannot read the division off the vertex of a parabola that was drawn symmetric by assumption.

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[52] John Rawls, A Theory of Justice 74 (Harvard Univ. Press rev. ed. 1999) ("A number of men are to divide a cake: assuming that the fair division is an equal one, which procedure, if any, will give this outcome? . . . [T]here is an independent criterion for what is a fair division, a criterion defined separately from and prior to the procedure which is to be followed.").

[53] Ken Binmore, Game Theory and the Social Contract: Just Playing 382–83 (MIT Press 1998).

[54] H. Peyton Young, Equity in Theory and Practice 135 (Princeton Univ. Press 1994).

[55] *See* Sidak, *The Fair Division of Surplus from a FRAND License Negotiated in Good Faith*, *supra* note 50, at 106–08 (deriving the even split from the symmetry of the bid and ask functions and from the cake-cutting protocol, and observing that no economic reason exists to expect such symmetry).

[56] *See* Ariel Rubinstein, *Perfect Equilibrium in a Bargaining Model*, 50 Econometrica 97 (1982); *see also* Sidak, *The Fair Division of Surplus from a FRAND License Negotiated in Good Faith*, *supra* note 50, at 107–08.

[57] Ken Binmore, Ariel Rubinstein & Asher Wolinsky, *The Nash Bargaining Solution in Economic Modelling*, 17 RAND J. Econ. 176 (1986).

D. The Right Reason

The rejection came for a narrower reason than it might have. The Federal Circuit in *VirnetX* did not say that the Nash bargaining solution conceals an interpersonal comparison. The court said only that the theorem asserts nothing about which real situations satisfy its premises. That ground was enough, because the expert had never shown that the Nash bargaining solution fit the record. The deeper reason—that the multiplication embeds an unproved equal-weighting judgment—lay ready to use, had anyone thought to raise it.

One eminent judge came nearer than the others. Striking a Nash-based damages report in *Oracle v. Google*, Judge William Alsup warned that the solution would "invite a miscarriage of justice by clothing a fifty-percent assumption in an impenetrable facade of mathematics."[58]

E. The Sufficiency of an Expert's Basis After EcoFactor

Eleven years separate *VirnetX* from *EcoFactor*, and in that interval the evidentiary law governing damages experts has been tightened twice—once by rule and once by the full Federal Circuit. The 2023 amendment to Rule 702 places upon the proponent of expert testimony the burden of demonstrating, more likely than not, that each condition of admissibility is satisfied, and the amended rule requires that the opinion reflect a reliable application of the expert's basis and methodology.[59] The Advisory Committee on Evidence Rules stated the occasion for the amendment without diplomacy: many courts had been treating the sufficiency of an expert's basis as a question of weight rather than of admissibility, and those rulings were an incorrect application of the rules.[60]

On May 21, 2025, in *EcoFactor, Inc. v. Google LLC*, the Federal Circuit sat *en banc* and applied that principle to a reasonable royalty.[61] The patent holder's damages expert testified that a per-unit rate was an established royalty, resting that conclusion upon three lump-sum settlement licenses.[62] The recitals of those licenses expressed only the patent holder's own belief that the lump sums had been computed from that rate, and two of them provided, in their operative payment provisions, that the amount was not based upon sales and

[58] Oracle Am., Inc. v. Google Inc., 798 F. Supp. 2d 1111, 1120 (N.D. Cal. 2011) (Alsup, J.) (citing Uniloc USA, Inc. v. Microsoft Corp., 632 F.3d 1292, 1312–15 (Fed. Cir. 2011) (Linn, J.)).

[59] Fed. R. Evid. 702 (2023).

[60] *Id.* advisory committee's note to 2023 amendment.

[61] EcoFactor, Inc. v. Google LLC, 137 F.4th 1333 (Fed. Cir. 2025) (*en banc*) (Moore, C.J.), *cert. denied*, 146 S. Ct. 333 (Oct. 20, 2025).

[62] *Id.* at 1341.

did not constitute a royalty.[63] The Federal Circuit held the testimony inadmissible and ordered a new trial on damages.[64]

Two strands of the court's reasoning matter here. First, Rule 702(b) directs attention to the facts and data upon which the expert purported to rely, not to the trial record at large. Second, the contested rate served as both the starting point and the outcome of the expert's hypothetical negotiation, so that when the premise failed, the opinion failed with it.[65]

The reader will recognize that shape, for it is the shape of the Nash-based damages opinion—a number supplied at the outset, adjusted by case-specific factors, and recovered, very nearly unchanged, at the end.

EcoFactor and the objection pressed here are tessellating complements—adjacent tiles rather than rival accounts of the same ground. *EcoFactor* asks whether the evidence upon which an expert relied will bear the premise that he asserted. The question here is the one logically anterior: whether the premise is of a kind that evidence could bear at all. When an expert derives a royalty from the Nash bargaining solution, the equal division is not an inference that has turned out to be poorly supported. The equal division is a property of the objective function that the expert selected. Multiply the parties' gains, and the maximum sits at the vertex; the vertex is symmetric because the function is symmetric; and the function is symmetric because the analyst chose it so.

No quantum of discovery could have improved that basis, because the basis is not evidentiary. The vice is not that the method is mathematical, but that the mathematics has been mistaken for a finding of fact. A rule of thumb does not cease to be a rule of thumb because it can be derived from an axiom; it becomes harder to see. The Supreme Court's warning in *Joiner*—that an opinion connected to the data only by the expert's own assertion need not be admitted—applies here *a fortiori*.[66] Here the opinion is not connected to the data at all at the decisive step. The opinion is connected to a multiplication sign.

That distinction also answers the strongest reply to pressing the point at the gatekeeping stage. Judge Leonard Stark, dissenting in part, warned that the majority risks converting Rule 702 into an instrument by which judges resolve disputes of fact under color of gatekeeping, and he invoked the Advisory Committee's instruction that the sufficiency inquiry does not authorize a trial court to believe one version of the facts rather than another.[67] Judge Stark is right about the danger, and his separate opinion, rather than the

[63] *Id.* at 1342–43.
[64] *Id.* at 1345–46.
[65] *Id.* at 1345–47.
[66] Gen. Elec. Co. v. Joiner, 522 U.S. 136, 146 (1997) (Rehnquist, C.J.).
[67] *EcoFactor*, 137 F.4th at 1355–56 (Stark, J., concurring in part and dissenting in part) (citing Micro Chem., Inc. v. Lextron, Inc., 317 F.3d 1387, 1392 (Fed. Cir. 2003) (Plager, J.)).

majority's, marks the true boundary of the doctrine. But the objection developed here lies wholly inside that boundary. There is no competing version of the facts under which the vertex of a parabola is read from evidence about the litigants. A court that excludes an even split derived from the symmetry axiom is not crediting one party's account of the record over the other's; it is observing that the number was never drawn from the record by either of them. Judge Stark and I are therefore describing one boundary from opposite sides. His dissent marks the cases in which gatekeeping trespasses upon the province of the jury. Nash's parabola marks a case in which gatekeeping plainly does not.

Conclusion

The Nash bargaining solution is an elegant result about the division of a fixed surplus between symmetric parties. It assigns equal weight to their utility gains, even though each party's utility is measurable only up to a separate positive affine transformation of his own. Where those conditions hold, the even split is a defensible focal point and the theorem a serviceable guide. Where they do not hold, the Nash bargaining solution measures the wrong thing, and does so with a confidence that its mathematical pedigree does not warrant. The conditions of the Nash bargaining solution fail whenever the parties are unequal in patience, in information, in risk preference, or in their outside options, or whenever the surplus itself is the creation whose value is disputed. Nash himself, in 1953, made the disagreement point depend upon the threats that each party could credibly deploy, and he thereby reopened the very questions of power and history that his 1950 bargaining solution had assumed away.

The bargain over a fixed pie was never the hard case. The hard case is the bargain over a pie that one of the parties must first bring into being, and for that case the multiplicative product of the shares is silent. The division must instead be inferred from the facts of the case—from the parties' outside options, their relative patience, and the prices struck in comparable bargains. That method does not multiply; it measures. I have developed it elsewhere.[68] Federal Rule of Evidence 702, as amended in 2023, and the Federal Circuit's decision in *EcoFactor* in 2025 now require the proponent of a Nash-derived royalty to establish the sufficiency of the basis on which his premise rests. For the equal division at the heart of the Nash bargaining parabola no such

[68] Sidak, *Bargaining Power and Patent Damages*, *supra* note 5, at 10–30 (developing the surplus-division principle); Sidak, *The Fair Division of Surplus from a FRAND License Negotiated in Good Faith*, *supra* note 50, at 96–108 (proposing and refining an efficiency rationale for why bargaining over patent royalties is typically observed in litigation, arbitration, or negotiation not to result in a 50-50 split of the surplus between the licensor and the licensee); J. Gregory Sidak, *What Makes FRAND Fair? The Just Price, Contract Formation, and the Division of Surplus from Voluntary Exchange*, 4 Criterion J. on Innovation 701 (2019).

basis exists, because the even split was never measured from the parties, their patience, or their alternatives.